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Orthogonal NOMA Weak Channel User Capacity: Effective Interference Suppression

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ABSTRACT

Recently, orthogonal non-orthogonal multiple access (O NOMA) with polar on-off keying (POOK) has been proposed to mitigate the severe effect of the superposition [6]. In [6], it is observed that the performance pattern of the O NOMA weak channel user is different from that of the standard NOMA. Such different pattern includes the effective inter user interference suppression and the better or worse performance for the parts of the whole range of power allocation. However, only the probability of errors is presented in [6] for the comparison of O NOMA to the standard NOMA and it is not clearly presented theoretically, with the ultimate bound, i.e., the channel capacity. This paper calculates the channel capacity of the O NOMA weak channel user and explains how the different pattern happens, based on the channel capacity.

Key Words : Non-orthogonal multiple access, successive interference cancellation, channel capacity, binary phase shift keying, power allocation

Ⅰ. Introduction

The fifth generation (5G) radio access networks require the huge number of mobile users to be served. There are no channel resources to accommodate such congestion. One of candidates to increase the system capacity is non-orthogonal multiple access (NOMA)[1-5] and the performance degradation of the superposition is almost resolved in orthogonal NOMA (O NOMA) with polar on-off keying (POOK)[6]. However, the proposed systems are analyzed only based on the probability of errors and the theoretical analysis is absent from [6]. This paper presents the channel capacity analysis for the O NOMA weak channel user and compares the calculated channel capacity to that of the standard NOMA [9]. The paper is organized as follows. Section II defines the system and channel model. In Section III, a brief review of POOK is summarized. In Section IV, the channel capacity is analyzed. In Section V, the results are presented and discussed. The paper is concluded in Section VI.

Ⅱ. System and Channel Model

Assume that the total transmit power is P, the power allocation factor is α with $0 \le \alpha \le 1$, ($0\% \le \alpha \le 100\%$), and the channel gains are h_1 and h_2 with $|h_1| > |h_2|$. Then αP is allocated to the user-1 signal s_1 and $(1-\alpha)P$ is allocated to the user-2 signal s_2, with $\mathbb{E}\left[|s_1|^2\right] = \mathbb{E}\left[|s_2|^2\right] = 1$. The expectation notation $\mathbb{E}[u]$ is defined as

$$\mathbb{E}[u] = \int_{-\infty}^{\infty} u p_U(u)\, du \tag{1}$$

where $p_U(u)$ is the probability density function (PDF). The superimposed signal is expressed by

$$x = \sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2 . \tag{2}$$

Before the successive interference cancellation (SIC) is performed on the user-1 with the better channel condition, the received signals of the user-1 and the user-2 are represented as

$$\begin{aligned} z_1 &= h_1\sqrt{\alpha P} s_1 + \left(h_1\sqrt{(1-\alpha)P} s_2 + w_1\right) \\ z_2 &= h_2\sqrt{(1-\alpha)P} s_2 + \left(h_2\sqrt{\alpha P} s_1 + w_2\right) \end{aligned} \tag{3}$$

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where w_1 and $w_2 \sim \mathcal{CN}(0, N_0)$ are complex additive white Gaussian noise (AWGN) and N_0 is one-sided power spectral density. The notation $\mathcal{CN}(\mu, \Sigma)$ denotes the complex circularly-symmetric normal distribution with mean μ and variance Σ. In addition, if the channel gains are assumed to be Rayleigh faded, then h_1 and $h_2 \sim \mathcal{CN}(0, 1^2)$. The coherent receivers of Rayleigh fading channels construct the following metrics from the received signals;

$$h_1^* z_1 = |h_1|^2 \sqrt{\alpha P} s_1 + \left(|h_1|^2 \sqrt{(1-\alpha)P} s_2 + h_1^* w_1 \right)$$
$$h_2^* z_2 = |h_2|^2 \sqrt{(1-\alpha)P} s_2 + \left(|h_2|^2 \sqrt{\alpha P} s_1 + h_2^* w_2 \right). \tag{4}$$

Furthermore, the receivers process the above metrics one step more;

$$\frac{h_1^*}{|h_1|} z_1 = |h_1| \sqrt{\alpha P} s_1 + \left(|h_1| \sqrt{(1-\alpha)P} s_2 + \frac{h_1^*}{|h_1|} w_1 \right)$$
$$\frac{h_2^*}{|h_2|} z_2 = |h_2| \sqrt{(1-\alpha)P} s_2 + \left(|h_2| \sqrt{\alpha P} s_1 + \frac{h_2^*}{|h_2|} w_2 \right). \tag{5}$$

Note that the noise $\frac{h_1^*}{|h_1|} w_1$ and $\frac{h_2^*}{|h_2|} w_2$ have the same statistics as w_1 and w_2, because $\frac{h_1^*}{|h_1|} = e^{j\theta}$ with θ uniformly distributed in $[0, \pi)$. Moreover, if the 1-dimensional modulation constellation is considered, the following metrics are sufficient statistics;

$$\begin{aligned} r_1 &= \mathrm{Re}\left\{ \frac{h_1^*}{|h_1|} z_1 \right\} \\ &= |h_1| \sqrt{\alpha P} s_1 + \left(|h_1| \sqrt{(1-\alpha)P} s_2 + \mathrm{Re}\left\{ \frac{h_1^*}{|h_1|} w_1 \right\} \right) \\ &= |h_1| \sqrt{\alpha P} s_1 + \left(|h_1| \sqrt{(1-\alpha)P} s_2 + n_1 \right) \\ r_2 &= \mathrm{Re}\left\{ \frac{h_2^*}{|h_2|} z_2 \right\} \\ &= |h_2| \sqrt{(1-\alpha)P} s_2 + \left(|h_2| \sqrt{\alpha P} s_1 + \mathrm{Re}\left\{ \frac{h_2^*}{|h_2|} w_2 \right\} \right) \\ &= |h_2| \sqrt{(1-\alpha)P} s_2 + \left(|h_2| \sqrt{\alpha P} s_1 + n_2 \right) \end{aligned} \tag{6}$$

where n_1 and $n_2 \sim \mathcal{N}(0, N_0 / 2)$ are additive white Gaussian noise (AWGN). The notation $\mathcal{N}(\mu, \Sigma)$ denotes the normal distribution with mean μ and variance Σ. In the standard NOMA, the SIC is performed only on the user-1. Then the received signal is given by, if the perfect SIC is assumed,

$$y_1 = r_1 - |h_1| \sqrt{(1-\alpha)P} s_2 = |h_1| \sqrt{\alpha P} s_1 + n_1. \tag{7}$$

Ⅲ. Brief Review of POOK

We, now, briefly review POOK [6]. On-off keying (OOK) is the simplest modulation technique. The carrier is sent or not. Assume the binary phase shift keying (BPSK) modulation for the user-1, with $s_1 \in \{+1, -1\}$. Then POOK, with $s_2 \in \{+\sqrt{2}, 0, -\sqrt{2}\}$, is the inter user interference s_1 dependent OOK. The power is normalized as

$$\mathbb{E}\left[|s_2|^2 \right] = \frac{1}{4}\left(+\sqrt{2}\right)^2 + \frac{1}{2}\left(\sqrt{0}\right)^2 + \frac{1}{4}\left(-\sqrt{2}\right)^2 = 1. \tag{8}$$

We compare POOK to the standard OOK, $s_{OOK} \in \{+\sqrt{2}, 0\}$ with

$$\mathbb{E}\left[|s_{OOK}|^2 \right] = \frac{1}{2}\left(\sqrt{2}\right)^2 + \frac{1}{2}\left(0\right)^2 = 1. \tag{9}$$

Remark that even though we compare POOK to OOK, it is not meant that POOK is considered as a variation of OOK. Since POOK has positive and negative polarities, it is closer to BPSK. If there exists the inter user interference s_1, polar OOK gets away from the interference in the direction from the origin to the interference. Therefore we give polarity to OOK, with the information input bits for the user-1 and the user-2 being $b_1, b_2 \in \{0, 1\}$, as

$$\begin{cases} s_1(b_1 = 0) = +1 \\ s_1(b_1 = 1) = -1 \end{cases}$$

$$\begin{cases} s_2(b_2 = 0 \mid b_1 = 0) = +\sqrt{2} \\ s_2(b_2 = 1 \mid b_1 = 0) = 0 \end{cases} \quad \begin{cases} s_2(b_2 = 0 \mid b_1 = 1) = -\sqrt{2} \\ s_2(b_2 = 1 \mid b_1 = 1) = 0. \end{cases} \tag{10}$$

Ⅳ. O NOMA Weak Channel User Capacity

In this paper, the channel capacity is defined as the maximum mutual information, maximized with an input PDF, without the shaping, (often called as the mutual information with equiprobable *M*-ary constellations[7]). The capacity of equiprobable *M*-ary constellations asymptotically approaches a straight line parallel to the capacity of the ideal Gaussian modulation[8], shifted right by $\pi e / 6$ (1.53 dB), which is the shaping loss. The capacity of equiprobable *M*-ary constellations saturates because information cannot be sent at a rate higher than $\log_2 M$. In O NOMA, it is interesting that the performance pattern of the O NOMA weak channel user is different from that of the standard NOMA [6]. Such different pattern includes the effective inter user interference suppression and the better or worse performance for the parts of the whole range of power allocation. We analyze such pattern with the channel capacity. The capacity in bit/s/Hz for the O NOMA weak channel user is calculated by

$$\begin{aligned} C_2^{(b;\ O\ NOMA)} &= \max_{p_{B_2}(b_2)} H(r_2) - H(r_2 \mid b_2) \\ &= -\int_{-\infty}^{\infty} p_{R_2}(r_2) \log_2 p_{R_2}(r_2) dr_2 \\ &+\int_{-\infty}^{\infty} p_{R_2|B_2}(r_2 \mid b_2 = 0) p_{B_2}(b_2 = 0) \log_2 p_{R_2|B_2}(r_2 \mid b_2 = 0) dr_2 \\ &+\int_{-\infty}^{\infty} p_{R_2|B_2}(r_2 \mid b_2 = 1) p_{B_2}(b_2 = 1) \log_2 p_{R_2|B_2}(r_2 \mid b_2 = 1) dr_2 \end{aligned} \tag{11}$$

where

$$\begin{aligned} p_{R_2|}(r_2) &= \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_2 - |h_2|\sqrt{\alpha P} - |h_2|\sqrt{2(1-\alpha)P}\right)^2}{2N_0/2}} \\ &+ \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_2 - |h_2|\sqrt{\alpha P}\right)^2}{2N_0/2}} \\ &+ \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_2 + |h_2|\sqrt{\alpha P}\right)^2}{2N_0/2}} \\ &+ \frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_2 + |h_2|\sqrt{\alpha P} + |h_2|\sqrt{2(1-\alpha)P}\right)^2}{2N_0/2}}. \end{aligned} \tag{12}$$

The conditional PDFs are given by

$$\begin{aligned} p_{R_2|B_2}(r_2 \mid b_2 = 0) &= \frac{1}{2} p_{R_2|B_1,B_2}(r_2 \mid b_1 = 0, b_2 = 0) \\ &+ \frac{1}{2} p_{R_2|B_1,B_2}(r_2 \mid b_1 = 1, b_2 = 0) \\ &= \frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_2 - |h_2|\sqrt{\alpha P} - |h_2|\sqrt{2(1-\alpha)P}\right)^2}{2N_0/2}} \\ &+ \frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_2 + |h_2|\sqrt{\alpha P} + |h_2|\sqrt{2(1-\alpha)P}\right)^2}{2N_0/2}} \end{aligned} \tag{13}$$

and

$$\begin{aligned} p_{R_2|B_2}(r_2 \mid b_2 = 1) &= \frac{1}{2} p_{R_2|B_1,B_2}(r_2 \mid b_1 = 0, b_2 = 1) \\ &+ \frac{1}{2} p_{R_2|B_1,B_2}(r_2 \mid b_1 = 1, b_2 = 1) \\ &= \frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_2 - |h_2|\sqrt{\alpha P}\right)^2}{2N_0/2}} \\ &+ \frac{1}{2}\frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{\left(r_2 + |h_2|\sqrt{\alpha P}\right)^2}{2N_0/2}}. \end{aligned} \tag{14}$$

Ⅴ. Results and Discussions

Assume that the channel gains are $|h_1| = 1.1$ and $|h_2| = 0.8$, and the total transmit signal power to one-sided power spectral density ratio is $P / N_0 = 10$, ($10\text{ dB} = 10\log_{10}(10)$). The channel capacities of the standard NOMA [9] and O NOMA in the equation (11) for the user-2 are shown in Fig. 1, with different power allocations, $0 \le \alpha \le 1$. We also include Table I for the numerical comparison of the capacities for some power allocation factors. As shown in Fig. 1, the effective inter user interference suppression is achieved at the vicinity of $\alpha = 50\%$, where the BPSK performance in the standard NOMA for the user-2 never improve, (The proof is found in [10] for the probability of errors). However, the O NOMA capacity is worse than the standard NOMA for about

$\alpha < 0.2$ or about $\alpha > 0.7$, because the O NOMA uses the part of the allocated power $(1-\alpha)P$ for the effective inter user interference suppression.

One of the advantages of O NOMA for the optimization of the power allocation is that in the standard NOMA, the capacity fluctuates over the entire power allocation range, especially around $\alpha = 0.5$ or $\alpha = 0.8$, while the O NOMA capacity changes linearly. Such advantage makes the power allocation easy in O NOMA.

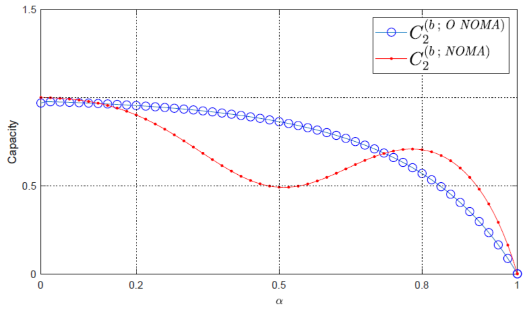

Fig. 1. Capacities of NOMA and O NOMA for the user-2.

Table 1. Capacity Comparison for Some Power Allocation Factors

NOMA	$\alpha=0.2$	$\alpha=0.5$	$\alpha=0.8$
orthogonal	0.9536	0.8622	0.5685
standard	0.9	0.4907	0.703

Ⅵ. Conclusion

First, we investigated the effective interference suppression of the O NOMA user-2 with the channel capacity, while in the previous researches, only the probability of errors was presented for the system comparison. It was also shown that the calculated channel capacity of O NOMA is better than that of the standard NOMA in the middle values of the power allocation factor. In result, the O NOMA could be considered according to the power allocation schemes.

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