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Impact of Fading on Performance of NOMA Weak Channel User

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ABSTRACT

We investigate the effect of fading on the performance of the weaker channel user in non-orthogonal multiple access (NOMA). This paper derives an analytical expression for the average performance of the NOMA weak channel user under Rayleigh fading channel. It is shown that the average performance of NOMA with the optimal maximum likelihood (ML) receiver is much closer to that of orthogonal multiple access (OMA), compared to that of NOMA with the standard receiver, except the vicinity of the power allocation factor 50%. In result, NOMA could be considered with the trade-off between the degraded performance and the increased system capacity.

Key Words : Non-orthogonal multiple access, successive interference cancellation, power allocation, Rayleigh fading, binary phase shift keyin

Ⅰ. Introduction

On April 3, 2019, for the first time in the world, the fifth generation (5G) mobile networks have been commercialized in Korea. However, the standardization for 5G and beyond mobile radio access networks is still in progress. One of 5G technologies is non-orthogonal multiple access (NOMA)[1-6]. In NOMA, multi users share the same channel resources for increasing the system capacity. Recently, the error probability of NOMA under Nakagami-*m* fading is presented in [7]. However, the error probability expression is conditioned on the transmitted signals of the other users and the error probability is calculated with the randomly chosen transmitted signals[7]. On the other hand, the researches for the achievable rate under fading channels have been presented for the sum-rate optimization[8] and for the asymptotic outage probability[9]. In [6], the performance of the NOMA weak channel user is presented for the fixed channel gain. However, in the practical mobile environments, channel fading should be considered. This paper derives the average performance of NOMA under Rayleigh fading and it is compared to that of orthogonal multiple access (OMA), such as time division multiple access (TDMA) and frequency division multiple access (FDMA). The paper is organized as follows. Section II defines the system and channel model. In Section III, the average performance is presented. In Section IV, the results are presented and discussed. The paper is concluded in Section V.

Ⅱ. System and Channel Model

Assume that the total transmit power is P, the power allocation factor is α with $0 \leq \alpha \leq 1$, ($0\% \leq \alpha \leq 100\%$), and the channel gains $h_1 \sim \mathcal{CN}(0,\Sigma_1)$ and $h_2 \sim \mathcal{CN}(0,\Sigma_2)$ are Rayleigh faded, with $\Sigma_1 > \Sigma_2$. The notation $\mathcal{CN}(\mu,\Sigma)$ denotes the complex circularly-symmetric normal distribution with mean μ and variance Σ. Then αP is allocated to the user-1 signal s_1 and $(1-\alpha)P$ is allocated to the user-2 signal s_2, with $\mathbb{E}\left[|s_1|^2\right] = \mathbb{E}\left[|s_2|^2\right] = 1$. The expectation notation $\mathbb{E}[u]$ is defined as

$$\mathbb{E}[u] = \int_{-\infty}^{\infty} u p_U(u)\, du \tag{1}$$

where $p_U(u)$ is the probability density function (PDF). The superimposed signal is expressed by

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$$x = \sqrt{\alpha P} s_1 + \sqrt{(1-\alpha)P} s_2 . \tag{2}$$

Before the successive interference cancellation (SIC) is performed on the user-1 with the better channel condition, the received signals of the user-1 and the user-2 are represented as

$$\begin{aligned} z_1 &= h_1 \sqrt{\alpha P} s_1 + \left(h_1 \sqrt{(1-\alpha)P} s_2 + w_1 \right) \\ z_2 &= h_2 \sqrt{(1-\alpha)P} s_2 + \left(h_2 \sqrt{\alpha P} s_1 + w_2 \right) \end{aligned} \tag{3}$$

where w_1 and $w_2 \sim \mathcal{CN}\left(0, N_0\right)$ are complex additive white Gaussian noise (AWGN) and N_0 is one-sided power spectral density. The coherent receivers of Rayleigh fading channels construct the following metrics from the received signals;

$$\begin{aligned} h_1^* z_1 &= \left|h_1\right|^2 \sqrt{\alpha P} s_1 + \left(\left|h_1\right|^2 \sqrt{(1-\alpha)P} s_2 + h_1^* w_1 \right) \\ h_2^* z_2 &= \left|h_2\right|^2 \sqrt{(1-\alpha)P} s_2 + \left(\left|h_2\right|^2 \sqrt{\alpha P} s_1 + h_2^* w_2 \right). \end{aligned} \tag{4}$$

Furthermore, the receivers process the above metrics one step more;

$$\begin{aligned} \frac{h_1^*}{\left|h_1\right|} z_1 &= \left|h_1\right| \sqrt{\alpha P} s_1 + \left(\left|h_1\right| \sqrt{(1-\alpha)P} s_2 + \frac{h_1^*}{\left|h_1\right|} w_1 \right) \\ \frac{h_2^*}{\left|h_2\right|} z_2 &= \left|h_2\right| \sqrt{(1-\alpha)P} s_2 + \left(\left|h_2\right| \sqrt{\alpha P} s_1 + \frac{h_2^*}{\left|h_2\right|} w_2 \right). \end{aligned} \tag{5}$$

Note that the noise $\frac{h_1^*}{\left|h_1\right|} w_1$ and $\frac{h_2^*}{\left|h_2\right|} w_2$ have the same statistics as w_1 and w_2, because $\frac{h_1^*}{\left|h_1\right|} = e^{j\theta}$ with θ uniformly distributed. Moreover, if the 1-dimensional modulation constellation is considered, the following metrics are sufficient statistics;

$$\begin{aligned} r_1 &= \mathrm{Re}\left\{ \frac{h_1^*}{\left|h_1\right|} z_1 \right\} \\ &= \left|h_1\right| \sqrt{\alpha P} s_1 + \left(\left|h_1\right| \sqrt{(1-\alpha)P} s_2 + \mathrm{Re}\left\{ \frac{h_1^*}{\left|h_1\right|} w_1 \right\} \right) \\ &= \left|h_1\right| \sqrt{\alpha P} s_1 + \left(\left|h_1\right| \sqrt{(1-\alpha)P} s_2 + n_1 \right) \\ r_2 &= \mathrm{Re}\left\{ \frac{h_2^*}{\left|h_2\right|} z_2 \right\} \\ &= \left|h_2\right| \sqrt{(1-\alpha)P} s_2 + \left(\left|h_2\right| \sqrt{\alpha P} s_1 + \mathrm{Re}\left\{ \frac{h_2^*}{\left|h_2\right|} w_2 \right\} \right) \\ &= \left|h_2\right| \sqrt{(1-\alpha)P} s_2 + \left(\left|h_2\right| \sqrt{\alpha P} s_1 + n_2 \right) \end{aligned} \tag{6}$$

where n_1 and $n_2 \sim \mathcal{N}\left(0, N_0 / 2\right)$ are additive white Gaussian noise (AWGN). The notation $\mathcal{N}\left(\mu, \Sigma\right)$ denotes the normal distribution with mean μ and variance Σ. In the standard NOMA, the SIC is performed only on the user-1. Then the received signal is given by, if the perfect SIC is assumed,

$$y_1 = r_1 - \left|h_1\right| \sqrt{(1-\alpha)P} s_2 = \left|h_1\right| \sqrt{\alpha P} s_1 + n_1 . \tag{7}$$

We assume the BPSK modulations for both users in the standard NOMA, i.e., the BPSK/BPSK NOMA,

$$s_1, s_2 \in \{+1, -1\}. \tag{8}$$

Ⅲ. Fading Performance Derivations

The conditional probability of errors $P_{e|h_2}^{(2;\, NOMA;\, ML;\, optimal)}$ for the user-2 with the maximum likelihood (ML) decoding over the weak channel h_2 is presented in [6]; for $\alpha < 0.5$,

$$\begin{aligned} & P_{e|h_2}^{(2;\, NOMA;\, ML;\, optimal)} \\ &= \frac{1}{2} Q\left(\frac{\left|h_2\right| \sqrt{P} \left(\sqrt{(1-\alpha)} + \sqrt{\alpha} \right)}{\sqrt{N_0 / 2}} \right) \\ &+ \frac{1}{2} Q\left(\frac{\left|h_2\right| \sqrt{P} \left(\sqrt{(1-\alpha)} - \sqrt{\alpha} \right)}{\sqrt{N_0 / 2}} \right) \end{aligned} \tag{9}$$

and for $\alpha > 0.5$,

$$\begin{aligned} & P_{e|h_2}^{(2;\, NOMA;\, ML;\, optimal)} \simeq \\ & + \frac{1}{2} \left(Q\left(\frac{\left|h_2\right| \sqrt{(1-\alpha)P}}{\sqrt{N_0 / 2}} \right) - Q\left(\frac{\left|h_2\right| \sqrt{P} \left(\sqrt{\alpha} + \sqrt{(1-\alpha)} \right)}{\sqrt{N_0 / 2}} \right) + Q\left(\frac{\left|h_2\right| \sqrt{P} \left(2\sqrt{\alpha} + \sqrt{(1-\alpha)} \right)}{\sqrt{N_0 / 2}} \right) \right) \\ & + \frac{1}{2} \left(Q\left(\frac{\left|h_2\right| \sqrt{(1-\alpha)P}}{\sqrt{N_0 / 2}} \right) + Q\left(\frac{\left|h_2\right| \sqrt{P} \left(\sqrt{\alpha} - \sqrt{(1-\alpha)} \right)}{\sqrt{N_0 / 2}} \right) - Q\left(\frac{\left|h_2\right| \sqrt{P} \left(2\sqrt{\alpha} - \sqrt{(1-\alpha)} \right)}{\sqrt{N_0 / 2}} \right) \right) \end{aligned} \tag{10}$$

where $Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$. Then the fading performance is calculated by

$$P_e^{(2;\, NOMA;\, ML;\, optimal)} = \mathbb{E}_{h_2}\left[P_{e|h_2}^{(2;\, NOMA;\, ML;\, optimal)} \right]. \tag{11}$$

Here, $Q\left(x\right)$ can be represented as

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{x^2}{2\sin^2\theta}} d\theta. \quad (12)$$

Thus we can use the well-known Rayleigh fading integration formula,

$$\int_0^{\infty} Q\left(\sqrt{2\gamma}\right)\frac{1}{\gamma_b} e^{-\frac{\gamma}{\gamma_b}}\, d\gamma = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_b}{1+\gamma_b}}\right) \quad (13)$$

where the random variable (RV) γ is exponentially distributed and the mean of γ is defined as

$$\gamma_b = \mathbb{E}\left[\gamma\right]. \quad (14)$$

Then the average probability of errors is calculated by, for $\alpha < 0.5$,

$$\begin{aligned} P_e^{(2;\,NOMA;\,ML;\,optimal)} &= \frac{1}{2}\cdot\frac{1}{2}\left(1 - \sqrt{\frac{\frac{P}{N_0}\left(\sqrt{(1-\alpha)}+\sqrt{\alpha}\right)^2\Sigma_2}{1+\frac{P}{N_0}\left(\sqrt{(1-\alpha)}+\sqrt{\alpha}\right)^2\Sigma_2}}\right) \\ &+\frac{1}{2}\cdot\frac{1}{2}\left(1 - \sqrt{\frac{\frac{P}{N_0}\left(\sqrt{(1-\alpha)}-\sqrt{\alpha}\right)^2\Sigma_2}{1+\frac{P}{N_0}\left(\sqrt{(1-\alpha)}-\sqrt{\alpha}\right)^2\Sigma_2}}\right) \end{aligned} \quad (15)$$

and for $\alpha > 0.5$,

$$\begin{aligned} P_e^{(2;\,NOMA;\,ML;\,optimal)} \simeq &+\frac{1}{2}\cdot\frac{1}{2}\left(\left(1-\sqrt{\frac{\frac{P}{N_0}(1-\alpha)\Sigma_2}{1+\frac{P}{N_0}(1-\alpha)\Sigma_2}}\right) - \left(1-\sqrt{\frac{\frac{P}{N_0}\left(\sqrt{\alpha}+\sqrt{(1-\alpha)}\right)^2\Sigma_2}{1+\frac{P}{N_0}\left(\sqrt{\alpha}+\sqrt{(1-\alpha)}\right)^2\Sigma_2}}\right) + \left(1-\sqrt{\frac{\frac{P}{N_0}\left(2\sqrt{\alpha}+\sqrt{(1-\alpha)}\right)^2\Sigma_2}{1+\frac{P}{N_0}\left(2\sqrt{\alpha}+\sqrt{(1-\alpha)}\right)^2\Sigma_2}}\right)\right) \\ &+\frac{1}{2}\cdot\frac{1}{2}\left(\left(1-\sqrt{\frac{\frac{P}{N_0}(1-\alpha)\Sigma_2}{1+\frac{P}{N_0}(1-\alpha)\Sigma_2}}\right) + \left(1-\sqrt{\frac{\frac{P}{N_0}\left(\sqrt{\alpha}-\sqrt{(1-\alpha)}\right)^2\Sigma_2}{1+\frac{P}{N_0}\left(\sqrt{\alpha}-\sqrt{(1-\alpha)}\right)^2\Sigma_2}}\right) - \left(1-\sqrt{\frac{\frac{P}{N_0}\left(2\sqrt{\alpha}-\sqrt{(1-\alpha)}\right)^2\Sigma_2}{1+\frac{P}{N_0}\left(2\sqrt{\alpha}-\sqrt{(1-\alpha)}\right)^2\Sigma_2}}\right)\right). \end{aligned} \quad (16)$$

Ⅳ. Results and Discussions

Assume that $\Sigma_2 = \left(0.9\right)^2$ and the total transmit signal power to one-sided power spectral density ratio is $P \,/\, N_0 = 40\ \mathrm{dB}$. We compare NOMA with the optimal ML detection in this paper to OMA, the average performance of which is given by

$$P_e^{(2;\,OMA)} = \frac{1}{2}\left(1 - \sqrt{\frac{\frac{P}{N_0}(1-\alpha)\Sigma_2}{1+\frac{P}{N_0}(1-\alpha)\Sigma_2}}\right). \quad (17)$$

In addition, we also compare NOMA with the optimal ML detection to the standard NOMA, in which the inter user interference is treated as Gaussian noise. In this case, the probability of errors is given by

$$P_e^{(2;\,NOMA;\,standard)} = \frac{1}{2}\left(1 - \sqrt{\frac{\frac{P(1-\alpha)\Sigma_2}{N_0+\alpha P\Sigma_2}}{1+\frac{P(1-\alpha)\Sigma_2}{N_0+\alpha P\Sigma_2}}}\right). \quad (18)$$

Then the probabilities of errors are compared in Fig. 1. We also show simulation results in Fig. 1, which are in good agreement with analytical results. As shown in Fig. 1, the performance of NOMA with the optimal ML detection is much better than that of the standard NOMA, because the optimal ML receiver exploits the statistical property of the inter user interference, which is disregarded in the standard receiver. The performance degradation of NOMA with the optimal ML detection over OMA is also observed. Remark that this paper does not try to show that NOMA performance is worse than OMA, because the channel capacity proves the fact, as follows,

$$\begin{aligned} C_2^{(OMA)} &= \log_2\left(1 + \frac{\left|h_2\right|^2(1-\alpha)P}{N_0}\right) \\ \geq C_2^{(NOMA)} &= \log_2\left(1 + \frac{\left|h_2\right|^2(1-\alpha)P}{\left|h_2\right|^2\alpha P + N_0}\right). \end{aligned} \quad (19)$$

The main contribution of this paper is that the performance degradation of NOMA over OMA can be decreased and the NOMA performance can be closer to the OMA performance, when the optimal ML receiver is used, instead of the standard receiver.

Note, however, that NOMA serves two users on the same channel resources, while OMA serves only a single user on the given channel resources. One comment on this paper is that NOMA operates on the user fairness principle, in which the weaker channel user with the more power faces with the stronger channel user with the less power; in that case, the inter user-2 interference, i.e., the weak power interference is practically ignored. This paper, however, analyzes the entire range of the power allocation. The authors are very careful for such analyses to mislead the readers to overlook the major NOMA principle. Nevertheless, the usefulness of the analysis for the entire range of the power allocation is that the results in this paper could be used for the power allocation, which can be one of the contributions of this paper.

It is meaningful to explain the analytical expression of this paper, for the intuitive perspective to the results in Fig. 1; first, for $\alpha < 0.5$, the dominant term in the equation (15) is given by

$$+\frac{1}{2}\cdot\frac{1}{2}\left(1-\sqrt{\frac{\frac{P}{N_0}\left(\sqrt{(1-\alpha)}-\sqrt{\alpha}\right)^2\Sigma_2}{1+\frac{P}{N_0}\left(\sqrt{(1-\alpha)}-\sqrt{\alpha}\right)^2\Sigma_2}}\right). \tag{20}$$

As α approaches 0.5, the dominant term of (20) becomes $\frac{1}{4}$, which is very close to the local maximum of the probability of errors. Second, for $\alpha > 0.5$ and toward $\alpha = 0.5$, the dominant term in the equation (16) is given by

$$+\frac{1}{2}\cdot\frac{1}{2}\left(1-\sqrt{\frac{\frac{P}{N_0}\left(\sqrt{\alpha}-\sqrt{(1-\alpha)}\right)^2\Sigma_2}{1+\frac{P}{N_0}\left(\sqrt{\alpha}-\sqrt{(1-\alpha)}\right)^2\Sigma_2}}\right). \tag{21}$$

As α approaches 0.5, the dominant term of (21) again becomes $\frac{1}{4}$. Third, for $\alpha > 0.5$ and toward $\alpha = 1$, the dominant term in the equation (16) is given by

$$+\frac{1}{2}\cdot\frac{1}{2}\left(1-\sqrt{\frac{\frac{P}{N_0}(1-\alpha)\Sigma_2}{1+\frac{P}{N_0}(1-\alpha)\Sigma_2}}\right). \tag{22}$$

As α approaches 1, the dominant term of (22) is the half of the probability of errors for OMA; note that we have the two identical dominant terms of (22). Then the probability of errors in the equation (16) approaches that of OMA exactly.

Now, we analyze the results in Fig. 1 more in detail; as shown in Fig. 1, the performance of NOMA with the optimal ML detection in this paper becomes closer to that of OMA, compared to that of the standard NOMA, except the vicinity of $\alpha = 0.5$. This observation suggests that if we avoid the power allocation of the vicinity of $\alpha = 0.5$, we could improve the NOMA performance significantly with the optimal ML detection, especially under Rayleigh fading channel environments. Then a natural question could be whether or not the NOMA performance can be improved at the vicinity of $\alpha = 0.5$; the topic will be an interesting research in the future.

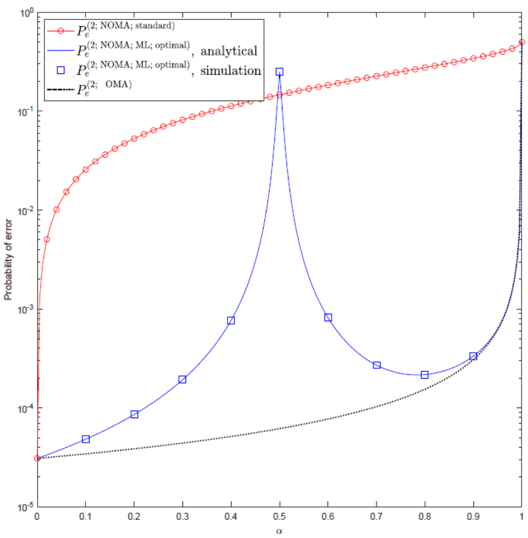

Fig. 1. Probabilities of errors for the standard NOMA, the optimal ML detection NOMA, and OMA for the user-2 under Rayleigh fading.

V. Conclusion

First we derived the fading performance of the weaker channel user in NOMA. Then we investigated the effect of the channel fading to NOMA systems.

It was shown how much the NOMA performance degrades, compared to the OMA. Consequently, there were gain and loss; the gain was that two users could use the same channel resources, i.e., the system capacity became double and the loss was the performance degradation, which was shown analytically in this paper.

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