# Optimal Detection for NOMA with Three Users 

Kyuhyuk Chung*


#### Abstract

It is straightforward to analyze the channel capacity for the arbitrary number of users in non-orthogonal multiple access (NOMA), due to the simple mathematical expression based on the ideal Gaussian modulation. However, under the practical modulation, such as binary phase shift keying (BPSK), it is not straightforward to analyze the performance for the entire range of the power allocation factor. Even with such difficulties, the performance has been analyzed for NOMA with two users, recently. In this paper, we challenge the performance analysis for NOMA with three users. It is shown for the entire coordinates of the power allocation factor pair that the performance of NOMA degrades with respect to that of orthogonal multiple access (OMA), such as time division multiple access (TDMA) and frequency division multiple access (FDMA). With such performance degradation, we triple the system capacity, because three users are served on the same channel resources. Therefore, a trade-off between the performance and the system capacity could be considered in NOMA.


Key Words : Non-orthogonal multiple access, successive interference cancellation, power allocation, channel capacity, binary phase shift keying.

## I. Introduction

Nowadays, 5G and beyond mobile networks, even 6G, are in the active researches, in step with 5 G communication commercialization in Korea, on April 3, 2019 for the first time in the world. One of such technologies is non-orthogonal multiple access
(NOMA) ${ }^{[1-5]}$. As opposed to the analysis of the channel capacity with the ideal Gaussian modulation in NOMA, the performance analysis based on the practical modulation, such as binary phase shift keying (BPSK), is not straightforward. Despite such difficulties of the analysis, the performance of NOMA with two users has been presented in [6]. The channel capacity of BPSK/BPSK NOMA is calculated in [7]. The probability of errors for the NOMA strong channel user without the successive interference cancellation (SIC) is derived in [8]. Also, the capacity for the non-SIC NOMA is calculated in [9]. The effect of the non-perfect SIC on the NOMA performance is investigated in [10]. In this paper, taking it one step further, we consider the performance analysis of NOMA with three users. Before we start the paper, it is valuable to mention the 3 -user limitation, instead of $N$-user; the ultimate goal is to analyze the NOMA performance with the arbitrary number of users. However, as opposed to the channel capacity analysis with Shannon capacity, $\log (1+P / N)$, which is relatively simple, the analysis based on the probability of errors is not straightforward for N -users. Therefore, we present the 3-user NOMA performance first, as an intermediate step. The paper is organized as follows. Section II defines the system and channel model. In Section III, the performance analysis is presented. In Section IV, the results are presented and discussed. The paper is concluded in Section V.

## II. System and Channel Model

Assume that the total transmit power is $P$, the power allocation factors are $\alpha$ and $\beta$ with $0 \leq \alpha, \beta \leq 1, \quad(0 \% \leq \alpha, \beta \leq 100 \%$; remark that the normal operating ranges are often $\alpha \leq 10 \%$ and $\beta \leq 20 \%$. However, in order for the analysis to be complete, we consider the entire coordinates of the power allocation factor pair. Such complete analyses give the broader perspective to the NOMA

- First Author : (ORCID:0000-0001-5429-2254)Department of Software Science, Dankook University, khchung@dankook.ac.kr, 종신회원
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performance.), and the channel gains are $h_{1}, h_{2}$ and $h_{3}$ with $\quad\left|h_{1}\right|>\left|h_{2}\right|>\left|h_{3}\right|$. Then $\alpha P, \quad \beta P \quad$ and $(1-\alpha-\beta) P$ are allocated to the user-1 signal $s_{1}$, the user-2 signal $s_{2}$, and the user- 3 signal $s_{3}$, respectively, with $\mathbb{E}\left[\left|s_{1}\right|^{2}\right]=\mathbb{E}\left[\left|s_{2}\right|^{2}\right]=\mathbb{E}\left[\left|s_{3}\right|^{2}\right]=1$. The superimposed signal is expressed by

$$
\begin{equation*}
x=\sqrt{\alpha P} s_{1}+\sqrt{\beta P} s_{2}+\sqrt{(1-\alpha-\beta) P} s_{3} . \tag{1}
\end{equation*}
$$

Before the SIC is performed on the user-1 and the user-2 with the better channel conditions, the received signals of the user- 1 , the user- 2 , and the user- 3 are represented as

$$
\begin{equation*}
r_{i}=\left|h_{i}\right| \sqrt{\alpha P} s_{1}+\left|h_{i}\right| \sqrt{\beta P} s_{2}+\left|h_{i}\right| \sqrt{(1-\alpha-\beta) P} s_{3}+n_{i} \tag{2}
\end{equation*}
$$

where $i=1,2,3 \quad$ and $\quad n_{i} \sim \mathcal{N}\left(0, N_{0} / 2\right) \quad$ are additive white Gaussian noise (AWGN). The notation $\boldsymbol{\mathcal { N }}(\mu, \Sigma)$ denotes the normal distribution with mean $\mu$ and variance $\Sigma$. In the standard NOMA, the SIC is performed only on the user-1 and the user- 2 . Then the received signal is given by, if the perfect SIC is assumed,

$$
\begin{align*}
& y_{1}=r_{1}-\left|h_{1}\right| \sqrt{\beta P} s_{2}-\left|h_{1}\right| \sqrt{(1-\alpha-\beta) P} s_{3}=\left|h_{1}\right| \sqrt{\alpha P} s_{1}+n_{1} \\
& y_{2}=r_{2}-\left|h_{2}\right| \sqrt{(1-\alpha-\beta) P} s_{3}=\left|h_{2}\right| \sqrt{\beta P} s_{2}+\left(\left|h_{2}\right| \sqrt{\alpha P} s_{1}+n_{2}\right)  \tag{3}\\
& y_{3}=r_{3} .
\end{align*}
$$

We assume the BPSK modulations for the three users in the standard NOMA, with $s_{1}, s_{2}, s_{3} \in\{+1,-1\}$.

## III. 3-User NOMA Performance Analysis

If the perfect SIC is assumed, then the probability of errors of the user- 1 is simply the performance of BPSK, for all $\alpha$,

$$
P_{e}^{(1 ; N O M A ; \text { perfect SIC } ; \text { ideal })}=Q\left(\left|h_{1}\right| \sqrt{\alpha P} / \sqrt{N_{0} / 2}\right)
$$

where $Q(x)=\int_{x}^{\infty} e^{-\frac{z^{2}}{2}} / \sqrt{2 \pi} d z$. Again if the perfect SIC is assumed, the probability of errors of the user-2 is simply the performance of the weak channel user in the two users $\mathrm{NOMA}^{[6]}$, with the proper changes of the parameters, for $\beta>\alpha$,

$$
\begin{equation*}
P_{e}^{(2 ; ~ N O M A ; \text {; peffect } S I C ; \text { ideal })}=\frac{1}{2} Q\left(\frac{\left|h_{2}\right| \sqrt{P}(\sqrt{\beta}+\sqrt{\alpha})}{\sqrt{N_{0} / 2}}\right)+\frac{1}{2} Q\left(\frac{\left|h_{2}\right| \sqrt{P}(\sqrt{\beta}-\sqrt{\alpha})}{\sqrt{N_{0} / 2}}\right) \tag{5}
\end{equation*}
$$

and for $\beta<\alpha$,

$$
\begin{align*}
& P_{e}^{(2 ; ~ N O M A ; ~ p e r f e c t ~ S I C ; ~ i d e a l) ~} \simeq \\
& +\frac{1}{2}\left\{Q\left(\frac{\left|h_{2}\right| \sqrt{\beta P}}{\sqrt{N_{0} / 2}}\right)-Q\left(\frac{\left|h_{2}\right| \sqrt{P}(\sqrt{\alpha}+\sqrt{\beta})}{\sqrt{N_{0} / 2}}\right)+Q\left(\frac{\left|h_{2}\right| \sqrt{P}(2 \sqrt{\alpha}+\sqrt{\beta})}{\sqrt{N_{0} / 2}}\right)\right\}  \tag{6}\\
& +\frac{1}{2}\left\{Q\left(\frac{\left|h_{2}\right| \sqrt{\beta P}}{\sqrt{N_{0} / 2}}\right)+Q\left(\frac{\left|h_{2}\right| \sqrt{P}(\sqrt{\alpha}-\sqrt{\beta})}{\sqrt{N_{0} / 2}}\right)-Q\left(\frac{\left|h_{2}\right| \sqrt{P}(2 \sqrt{\alpha}-\sqrt{\beta})}{\sqrt{N_{0} / 2}}\right)\right\} .
\end{align*}
$$

Now, we derive an analytical expression on the performance of the optimal receiver for the user-3. The maximum likelihood (ML) detection is made as

$$
\begin{equation*}
\hat{s}_{3}=\underset{s_{3} \in\{+1,-1\}}{\arg \max } p_{R_{3} \mid S_{3}}\left(r_{3} \mid s_{3}\right) \tag{7}
\end{equation*}
$$

The likelihood $p_{R_{3} \mid S_{3}}\left(r_{3} \mid s_{3}\right)$ is expressed by

$$
\begin{align*}
& p_{R_{3} \mid S_{3}}\left(r_{3} \mid s_{3}\right) \\
& =\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} \sum_{i=0}^{1} \sum_{j=0}^{1} e^{-\frac{\left(r_{3}+(-1)^{3}\left|h_{3}\right| \sqrt{\alpha P}+(-1)^{j}\left|h_{1}\right| \sqrt{\beta P}-\left|n_{3}\right| \sqrt{(1-\alpha-\beta) P_{3}}\right)^{2}}{2 N_{0} / 2}} . \tag{8}
\end{align*}
$$

We consider the sum $\alpha+\beta$ of the power allocation factors less than $1 / 3$. In this case, there is the exact single decision boundary, $r_{3}=0$. Then for $\alpha+\beta<1 / 3$, for $s_{1}=+1$, the decision region is $r_{3}>0$. Then the probability of errors is calculated by, for $\alpha+\beta<1 / 3$,

$$
\begin{equation*}
P_{e}^{(3 ; \text { NOMA; ML; optimal) }}=q^{(+1 ;+1 ;+1)}+q^{(+1 ;+1 ;-1)}+q^{(+1 ;-1 ;+1)}+q^{(-1 ;-1 ;-1)} \tag{9}
\end{equation*}
$$

where for the simplification, we define the notation

$$
\begin{equation*}
q^{(S ; B ; A)}=Q\left(S\left|h_{3}\right| \sqrt{P}(\sqrt{1-\alpha-\beta}+B \sqrt{\beta}+A \sqrt{\alpha}) / \sqrt{N_{0} / 2}\right) / 4 \tag{10}
\end{equation*}
$$

For $1 / 3<\alpha+\beta<0.5$, the decision boundaries are dependent on $\alpha$. Specifically, the exact single decision boundary is $r_{3}=0$, for the following intervals.
$\alpha<(\alpha+\beta)-\sqrt{(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}} / 2, \alpha>\left((\alpha+\beta)+\sqrt{\left.(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}\right)} / 2\right.$.
In this case, the probability of errors is the same as that in the equation (9) . However, for the following interval,

$$
\begin{equation*}
\left((\alpha+\beta)-\sqrt{(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}}\right) / 2<\alpha<(\alpha+\beta) / 2 \tag{12}
\end{equation*}
$$

there are one exact decision boundary and two approximate boundaries,

$$
\begin{equation*}
r_{3}=0, \quad r_{3} \simeq \pm\left|h_{3}\right| \sqrt{\alpha P} \tag{13}
\end{equation*}
$$

The approximate calculation error is small and tolerable, resorting to the $68-95-99.7$ rule, for $\boldsymbol{\mathcal { N }}\left(0,1^{2}\right)$. For this case, the probability of errors is calculated by
$P_{e}^{(3 ; \text { NOMA; ML; optimal })}=$
$+q^{(+1 ;+1 ; 0)}-q^{(+1 ;+1 ;+1)}+q^{(+1 ;+1 ;+2)}+q^{(+1 ;+1 ;-2)}-q^{(+1 ;+1 ;-1)}+q^{(+1 ;+1 ; 0)}$
$+q^{(+1 ;-1 ; 0)}-q^{(+1 ;-1 ;+1)}+q^{(+1 ;-1 ;+2)}-q^{(-1 ;-1 ;-2)}+q^{(-1 ;-1 ;-1)}+q^{(+1 ;-1 ; 0)}$
In addition, for the following interval,

$$
\begin{equation*}
(\alpha+\beta) / 2<\alpha<\left((\alpha+\beta)+\sqrt{(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}}\right) / 2 \tag{15}
\end{equation*}
$$

there are one exact decision boundary and two approximate boundaries,

$$
\begin{equation*}
r_{3}=0, \quad r_{3} \simeq \pm\left|h_{3}\right| \sqrt{\beta P} \tag{16}
\end{equation*}
$$

For this case, the probability of errors is calculated by interchanging $\alpha$ and $\beta$ in the equation (14).

For $0.5<\alpha+\beta<2 / 3$, the decision boundaries are dependent on $\alpha$, too. Specifically, for the following interval,

$$
\begin{equation*}
0<\alpha<\left((\alpha+\beta)-\sqrt{(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}}\right) / 2 \tag{17}
\end{equation*}
$$

the decision boundaries are the same as those in the equation (16) . For this case, the probability of errors is calculated by

$$
\begin{align*}
& P_{e}^{(3 ; \text { NOMA; ML; optimal })}= \\
& +q^{(+1 ; 0 ;+1)}-q^{(+1 ;+1 ;+1)}+q^{(+1 ;+2 ;+1)}+q^{(+1 ; 0 ;-1)}-q^{(+1 ;+1 ;-1)}+q^{(+1 ;+2 ;-1)}  \tag{18}\\
& -q^{(-1 ;-2 ;+1)}+q^{(-1 ;-1 ;+1)}+q^{(+1 ; 0 ;+1)}-q^{(-1 ;-2 ;-1)}+q^{(-1 ;-1 ;-1)}+q^{(+1 ; 0 ;-1)} .
\end{align*}
$$

In addition, for $0.5<\alpha+\beta<2 / 3$, for the following interval,

$$
\begin{equation*}
\left((\alpha+\beta)+\sqrt{(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}}\right) / 2<\alpha<(\alpha+\beta) \tag{19}
\end{equation*}
$$

the decision boundaries are the same as those in the equation (13) .

For this case, the probability of errors is calculated by interchanging $\alpha$ and $\beta$ in the equation (18). Continuingly, for $0.5<\alpha+\beta<2 / 3$, for the following interval,

$$
\begin{equation*}
2(\alpha+\beta)-1<\alpha<(\alpha+\beta) / 2 \tag{20}
\end{equation*}
$$

the decision boundaries are the same as those in the equation (13), and the probability of errors is the same as that in the equation (14). Also, for $0.5<\alpha+\beta<2 / 3$, for the following interval,

$$
\begin{equation*}
(\alpha+\beta) / 2<\alpha<1-(\alpha+\beta) \tag{21}
\end{equation*}
$$

the decision boundaries are the same as those in the equation (16), and the probability of errors is calculated by interchanging $\alpha$ and $\beta$ in the equation (14) . Continuingly, for $0.5<\alpha+\beta<2 / 3$, for the following interval,

$$
\begin{equation*}
\left((\alpha+\beta)-\sqrt{(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}}\right) / 2<\alpha<2(\alpha+\beta)-1 \tag{22}
\end{equation*}
$$

there are one exact decision boundary and four approximate boundaries,

$$
\begin{equation*}
r_{3}=0, \quad r_{3} \simeq \pm\left|h_{3}\right| \sqrt{\alpha P}, \quad r_{3} \simeq \pm\left|h_{3}\right| \sqrt{\beta P} \tag{23}
\end{equation*}
$$

For this case, the probability of errors is calculated by

$$
\begin{align*}
& P_{e}^{(3 ; \text { NOMA; ML; optimal })}= \\
& +q^{(+1 ; 0 ;+1)}-q^{(+1 ;+1 ; 0)}+q^{(+1 ;+1 ;+1)}-q^{(+1 ;+1 ;+2)}+q^{(+1 ;+2 ;+1)} \\
& +q^{(+1 ; 0 ;-1)}-q^{(+1 ;+1 ;-2)}+q^{(+1 ;+1 ;-1)}-q^{(+1 ;+1 ; 0)}+q^{(+1 ;+2 ;-1)}  \tag{24}\\
& -q^{(-1 ;-2 ;+1)}+q^{(-1 ;-1 ; 0)}+q^{(+1 ;-1 ;+1)}-q^{(+1 ;-1 ;+2)}+q^{(+1 ; 0 ;+1)} \\
& -q^{(-1 ;-2 ;-1)}+q^{(-1 ;-1 ;-2)}-q^{(-1 ;-1 ;-1)}+q^{(-1 ;-1 ; 0)}+q^{(+1 ; 0 ;-1)}
\end{align*}
$$

Continuingly, for $0.5<\alpha+\beta<2 / 3$, for the following interval,

$$
\begin{equation*}
1-(\alpha+\beta)<\alpha<\left((\alpha+\beta)+\sqrt{(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}}\right) / 2 \tag{25}
\end{equation*}
$$

the decision boundaries are the same as those in the equation (23) . For this case, the probability of errors is calculated by interchanging $\alpha$ and $\beta$ in the equation (24) . Next, for $2 / 3<\alpha+\beta<5 / 6$, for the following interval,

$$
\begin{equation*}
0<\alpha<\left((\alpha+\beta)-\sqrt{(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}}\right) / 2 \tag{26}
\end{equation*}
$$

the decision boundaries are the same as those in the equation (16), and the probability of errors is the same as that in the equation (18), and for the following interval,

$$
\begin{equation*}
\left((\alpha+\beta)+\sqrt{(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}}\right) / 2<\alpha<(\alpha+\beta) \tag{27}
\end{equation*}
$$

the decision boundaries are the same as those in the equation (13), and the probability of errors is calculated by interchanging $\alpha$ and $\beta$ in the equation (18) . Continuingly, for $2 / 3<\alpha+\beta<5 / 6$,
$\left((\alpha+\beta)-\sqrt{(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}}\right) / 2<\alpha<1-(\alpha+\beta)$
the decision boundaries are the same as those in the equation (23), and the probability of errors is the same as that in the equation (24), and for the following interval,

$$
\begin{equation*}
2(\alpha+\beta)-1<\alpha<\left((\alpha+\beta)+\sqrt{(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}}\right) / 2 \tag{29}
\end{equation*}
$$

the decision boundaries are the same as those in the equation (23), and the probability of errors is calculated by interchanging $\alpha$ and $\beta$ in the equation (24) . Continuingly, for $2 / 3<\alpha+\beta<5 / 6$,

$$
\begin{equation*}
1-(\alpha+\beta)<\alpha<(\alpha+\beta) / 2 \tag{30}
\end{equation*}
$$

there are one exact decision boundary and four approximate boundaries,

$$
\begin{equation*}
r_{3}=0, \quad r_{3} \simeq \pm\left|h_{3}\right| \sqrt{P}(\sqrt{\alpha}+\sqrt{\beta}), \quad r_{3} \simeq \pm\left|h_{3}\right| \sqrt{\beta P} \tag{31}
\end{equation*}
$$

For this case, the probability of errors is calculated by

$$
\begin{aligned}
& P_{e}^{(3 ; \text { NOMA; ML; optimal })}= \\
& +q^{(+1 ; ; 0 ; 0)}-q^{(+1 ; 0 ;+1)}+q^{(+1 ;+1 ;+1)}-q^{(+1 ;+2 ;+1)}+q^{(+1 ;+2 ;+2)} \\
& -q^{(-1 ; 0 ;-2)}+q^{(-1 ; 0 ;-1)}+q^{(+1 ;+1 ;-1)}-q^{(+1 ;+2 ;-1)}+q^{(+1 ;+2 ; 0)} \\
& -q^{(-1 ;-2 ; 0)}+q^{(-1 ;-2 ;+1)}+q^{(+1 ;-1 ;+1)}-q^{(+1 ; 0 ;+1)}+q^{(+1 ; 0 ;+2)} \\
& -q^{(-1 ;-2 ;-2)}+q^{(-1 ;-2 ;-1)}-q^{(-1 ;-1 ;-1)}+q^{(-1 ; 0 ;-1)}+q^{(+1 ; 0 ; 0)}
\end{aligned}
$$

Continuingly, for $2 / 3<\alpha+\beta<5 / 6$,

$$
\begin{equation*}
(\alpha+\beta) / 2<\alpha<2(\alpha+\beta)-1 \tag{33}
\end{equation*}
$$

there are one exact decision boundary and four approximate boundaries,

$$
\begin{equation*}
r_{3}=0, \quad r_{3} \simeq \pm\left|h_{3}\right| \sqrt{P}(\sqrt{\alpha}+\sqrt{\beta}), \quad r_{3} \simeq \pm\left|h_{3}\right| \sqrt{\alpha P} \tag{34}
\end{equation*}
$$

For this case, the probability of errors is calculated by interchanging $\alpha$ and $\beta$ in the equation (32).

Lastly, for $5 / 6<\alpha+\beta<1$,

$$
\begin{equation*}
0<\alpha<1-(\alpha+\beta) \tag{35}
\end{equation*}
$$

the decision boundaries are the same as those in the equation (16) , and the probability of errors is the same as that in the equation (18), and for

$$
\begin{equation*}
2(\alpha+\beta)-1<\alpha<(\alpha+\beta) \tag{36}
\end{equation*}
$$

the decision boundaries are the same as those in the equation (13) , and the probability of errors is calculated by interchanging $\alpha$ and $\beta$ in the equation (18) . Continuingly, for $5 / 6<\alpha+\beta<1$, for the following interval,

$$
\begin{equation*}
\left((\alpha+\beta)-\sqrt{(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}}\right) / 2<\alpha<(\alpha+\beta) / 2 \tag{37}
\end{equation*}
$$

the decision boundaries are the same as those in the equation (31), and the probability of errors is the same as that in the equation (32), and for the following interval,

$$
\begin{equation*}
(\alpha+\beta) / 2<\alpha<\left((\alpha+\beta)-\sqrt{(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}}\right) / 2 \tag{38}
\end{equation*}
$$

the decision boundaries are the same as those in the equation (34) , and the probability of errors is calculated by interchanging $\alpha$ and $\beta$ in the equation (32) . Continuingly, for $5 / 6<\alpha+\beta<1$, for the following interval,

$$
\begin{equation*}
1-(\alpha+\beta)<\alpha<\left((\alpha+\beta)-\sqrt{(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}}\right) / 2 \tag{39}
\end{equation*}
$$

there are one exact decision boundary and six approximate boundaries,
$r_{3}=0, \quad r_{3} \simeq \pm\left|h_{3}\right| \sqrt{P}(\sqrt{\alpha}+\sqrt{\beta}), \quad r_{3} \simeq \pm\left|h_{3}\right| \sqrt{P}(-\sqrt{\alpha}+\sqrt{\beta}), \quad r_{3} \simeq \pm\left|h_{3}\right| \sqrt{\beta P}$.
For this case, the probability of errors is calculated by

```
P
+q}\mp@subsup{q}{}{(+1;:0)}-\mp@subsup{q}{}{(+1;;+1)}+\mp@subsup{q}{}{(+1;;;+2)}-\mp@subsup{q}{}{(+1;+1;+1)}+\mp@subsup{q}{}{(+1;+;;0)}-\mp@subsup{q}{}{(+1;+2;+1)}+\mp@subsup{q}{}{(+1;+2;+2)
-q}\mp@subsup{q}{(-1;;;-2)}{(-\mp@subsup{q}{}{(-1;;;-1)}+\mp@subsup{q}{}{(+1;;0)}-\mp@subsup{q}{}{(+1;+1;-1)}+\mp@subsup{q}{}{(+1;+2;-2)}-\mp@subsup{q}{}{(+1;+2;-1)}+\mp@subsup{q}{}{(+1;+;;0)}
-q}\mp@subsup{q}{}{(-1;-2;0)}+\mp@subsup{q}{}{(-1;-2;+1)}-\mp@subsup{q}{}{(-1;-2;2)}+\mp@subsup{q}{}{(-1;-1;+1)}+\mp@subsup{q}{}{(+;;;;0)}-\mp@subsup{q}{}{(+1;;;+1)}+\mp@subsup{q}{}{(+1;;+2)
-q}\mp@subsup{q}{}{(-1;-2;-2)}+\mp@subsup{q}{}{(-1;-2;-1)}-\mp@subsup{q}{}{(-1;-2:0)}+\mp@subsup{q}{}{(-1;-1;-1)}-\mp@subsup{q}{}{(-1;0;-2)}+\mp@subsup{q}{}{(-1;;;-1)}+\mp@subsup{q}{}{(+1;:0;)
```

Continuingly, for $5 / 6<\alpha+\beta<1$, for the following interval,

$$
\begin{equation*}
\left((\alpha+\beta)+\sqrt{(\alpha+\beta)^{2}-(2(\alpha+\beta)-1)^{2}}\right) / 2<\alpha<2(\alpha+\beta)-1 \tag{42}
\end{equation*}
$$

there are one exact decision boundary and six approximate boundaries,

$$
\begin{equation*}
r_{3}=0, \quad r_{3} \simeq \pm\left|n_{3}\right| \sqrt{P}(\sqrt{\alpha}+\sqrt{\beta}), \quad r_{3} \simeq \pm\left|r_{3}\right| \sqrt{P}(\sqrt{\alpha}-\sqrt{\beta}), \quad r_{3} \simeq \pm\left|n_{3}\right| \sqrt{\beta P} . \tag{43}
\end{equation*}
$$

For this case, the probability of errors is calculated by interchanging $\alpha$ and $\beta$ in the equation (41).

## IV. Results and Discussions

Assume that the channel gain is $\left|h_{3}\right|=0.8$ and the total transmit signal power to one-sided power spectral density ratio is $P / N_{0}=20 \mathrm{~dB}$. We compare NOMA to orthogonal multiple access (OMA), the performance of which is given by

$$
\begin{equation*}
P_{e}^{(3 ; O M A)}=Q\left(\left|h_{3}\right| \sqrt{(1-\alpha-\beta) P} / \sqrt{N_{0} / 2}\right) \tag{44}
\end{equation*}
$$

As shown in Fig. 1, the result is consistent with the NOMA principle that the more power is allocated to the weaker channel user; for $\alpha \simeq 10 \%$ and


Fig. 1. Probabilities of errors with NOMA and OMA for the user-3.
$\beta \simeq 20 \%$, the performance of NOMA is very close to that of OMA, for the user-3.

## V. Conclusion

This paper analyzed the NOMA performance with three users. Especially, the analysis was complete in that the probability of errors was calculated for the whole coordinates of the power allocation factor pair. In NOMA of three users, we could triple the system capacity, at the price of the performance degradation, which was investigated in this paper. As a result, NOMA systems could be designed with the consideration of such performance degradation.

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