

How Can Near-Perfect SIC NOMA Be Implemented? (Part II): Complement Superposition Coding for 4-Ary Pulse Amplitude Modulation

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ABSTRACT

In non-orthogonal multiple access (NOMA), the successive interference cancellation (SIC) implementation is very important, because the performance of the stronger channel user with the low power allocation is greatly affected by the SIC performance. Up to now, in order to achieve the perfect SIC performance, two techniques for the single level modulation, such as binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK), have been reported in the literature; one way is the dynamic superposition coding (DSC), which is the combination of the normal superposition coding (NSC) and symmetric superposition coding (SSC). The other way is the orthogonal NOMA with polar on-off keying (POOK). As an extended research, this paper challenges the perfect SIC NOMA performance for the 4-ary pulse amplitude modulation (4PAM); the perfect SIC performance cannot be obtained by a simple combination of NSC and SSC, just as for BPSK. For 4PAM case, we need new superposition coding, i.e., the complement superposition coding (CSC). It is shown analytically that the combination of NSC, SSC, and CSC can achieve the near-perfect SIC performance. As a result, the DSC concept could be applied for the multilevel modulations, in order to achieve the near-perfect SIC performance in NOMA.

Key Words : NOMA, successive interference cancellation, power allocation, channel capacity, binary phase shift

keying, 4-ary pulse amplitude modulation

I. Introduction

NON-ORTHOGONAL multiple access (NOMA) has been proposed as one of the promising techniques for the fifth generation (5G) mobile communication^[1-6]. In NOMA, the successive interference cancellation (SIC) implementation greatly affects the performance of the stronger channel user. Thus, for the simplest modulation of the single level modulations, such as binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK), we presented the achievable rate region and the optimal maximum likelihood (ML) bit-error rate (BER) performance^[7-9]. The key contribution of [7-9] is as follows; even though Gaussian modulations achieve the channel capacity, at the same time, if such Gaussian modulations are treated as the inter user interference (IUI), that noise is the worst noise; thus the optimal ML receiver considers IUI as the practical modulation, such as BPSK, i.e., BPSK IUI, not Gaussian IUI. The fixed channel gain model is gradually replaced by the more practical channel model, i.e., fading channel model^[10-12]; even though three works have been done independently by different authors, there are different contributions: in [10], IUI is still Gaussian. In [11], the analytical expressions are derived only for the partial power allocation factor range $0\% \leq \alpha \leq 50\%$. In [12], the optimal ML performance is presented for all power allocation $0\% \leq \alpha \leq 100\%$. After the optimal performance has been analyzed for the single level modulations, the efforts to achieve the perfect SIC performance have been reported with the symmetric superposition coding (SSC)^[13-15], the dynamic superposition coding (DSC)^[16] and polar on-off keying (POOK)^[17-19]. (And even though the details can be found therein, we summarize shortly; SSC BPSK NOMA^[13-15] achieves the perfect SIC performance only for

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$0\% \leq \alpha \leq 50\%$; DSC BPSK NOMA achieves the near-perfect SIC performance for all $0\% \leq \alpha \leq 100\%$; the BPSK/POOK NOMA achieves the perfect SIC performance for the entire range $0\% \leq \alpha \leq 100\%$.) Now, we observe what is between the ideal infinite number of levels and a practical single level for modulations. The obvious step from a single level to the infinite level should be the multilevel modulations. Among them, the simplest one is 4-ary pulse amplitude modulation (4PAM). By the similar roadmap for the single level modulation, first the optimal performance has been analyzed^[20,21]. And such the performance has been improved by the SSC technique^[22]. Then this paper tries to achieve the near-perfect SIC performance. We choose the DSC approach. (The other way with POOK currently looks intractable for multilevel modulations, i.e., the 4PAM/4PAM NOMA, even though the polarity modulation has been invented for the BPSK/4PAM NOMA^[19], so that the more research efforts for POOK approach are required for the future.) We present the DSC with the existing the normal superposition coding (NSC) and SCC, including the complement superposition coding (CSC), which is proposed in this paper, for the 4PAM/4PAM NOMA. It is shown analytically that DSC(NSC, SSC, CSC) 4PAM/4PAM NOMA achieves the near-perfect SIC performance, where DSC(.,.) is defined as a combination of the given superposition coding schemes. In addition, we summarize the superposition coding schemes; The NSC is based on the natural order of the signal mapping. The SSC is characterized by the symmetric signal mapping. And the CSC is designed for maximizing the signal point distances.

Lastly, we summarize our contributions, in order to clarify the new contributions of this paper;

- Our contributions: 1. The CSC for 4PAM/4PAM NOMA is proposed. 2. The performance of the proposed scheme is analyzed. 3. It is shown that the proposed system achieves the near-perfect SIC performance.

II. System and Channel Model

Assume that the total transmit power is P , the power allocation factor is α with $0 \leq \alpha \leq 1$, ($0\% \leq \alpha \leq 100\%$), and the channel gains $h_1 \sim \mathcal{CN}(0, \Sigma_1)$ and $h_2 \sim \mathcal{CN}(0, \Sigma_2)$ are Rayleigh faded, with $\Sigma_1 > \Sigma_2$. The notation $\mathcal{CN}(\mu, \Sigma)$ denotes the complex circularly-symmetric normal distribution with mean μ and variance Σ . Then αP is allocated to the user-1 signal s_1 and $(1 - \alpha)P$ is allocated to the user-2 signal s_2 , with $\mathbb{E}[|s_1|^2] = \mathbb{E}[|s_2|^2] = 1$. The superimposed signal is expressed by

$$x = \sqrt{\alpha P} s_1 + \sqrt{(1 - \alpha)P} s_2. \quad (1)$$

Before the successive interference cancellation (SIC) is performed on the user-1 with the better channel condition, the received signals of the user-1 and the user-2 are represented as

$$\begin{aligned} z_1 &= h_1 \sqrt{\alpha P} s_1 + \left(h_1 \sqrt{(1 - \alpha)P} s_2 + w_1 \right) \\ z_2 &= h_2 \sqrt{(1 - \alpha)P} s_2 + \left(h_2 \sqrt{\alpha P} s_1 + w_2 \right) \end{aligned} \quad (2)$$

where w_1 and $w_2 \sim \mathcal{CN}(0, N_0)$ are complex additive white Gaussian noise (AWGN) and N_0 is one-sided power spectral density. The coherent receivers of Rayleigh fading channels construct the following metrics from the received signals;

$$\begin{aligned} h_1^* z_1 &= |h_1|^2 \sqrt{\alpha P} s_1 + \left(|h_1|^2 \sqrt{(1 - \alpha)P} s_2 + h_1^* w_1 \right) \\ h_2^* z_2 &= |h_2|^2 \sqrt{(1 - \alpha)P} s_2 + \left(|h_2|^2 \sqrt{\alpha P} s_1 + h_2^* w_2 \right). \end{aligned} \quad (3)$$

Furthermore, the receivers process the above metrics one step more;

$$\begin{aligned} \frac{h_1^*}{|h_1|} z_1 &= |h_1| \sqrt{\alpha P s_1} + \left(|h_1| \sqrt{(1-\alpha) P s_2} + \frac{h_1^*}{|h_1|} w_1 \right) \\ \frac{h_2^*}{|h_2|} z_2 &= |h_2| \sqrt{(1-\alpha) P s_2} + \left(|h_2| \sqrt{\alpha P s_1} + \frac{h_2^*}{|h_2|} w_2 \right). \end{aligned} \quad (4)$$

Note that the noise $\frac{h_1^*}{|h_1|} w_1$ and $\frac{h_2^*}{|h_2|} w_2$ have the same statistics as w_1 and w_2 , because $\frac{h_1^*}{|h_1|} = e^{j\theta}$ with θ uniformly distributed. Moreover, if the 1-dimensional modulation constellation is considered, the following metrics are sufficient statistics;

$$\begin{aligned} r_1 &= \text{Re} \left\{ \frac{h_1^*}{|h_1|} z_1 \right\} \\ &= |h_1| \sqrt{\alpha P s_1} + \left(|h_1| \sqrt{(1-\alpha) P s_2} + \text{Re} \left\{ \frac{h_1^*}{|h_1|} w_1 \right\} \right) \\ &= |h_1| \sqrt{\alpha P s_1} + \left(|h_1| \sqrt{(1-\alpha) P s_2} + n_1 \right) \\ r_2 &= \text{Re} \left\{ \frac{h_2^*}{|h_2|} z_2 \right\} \\ &= |h_2| \sqrt{(1-\alpha) P s_2} + \left(|h_2| \sqrt{\alpha P s_1} + \text{Re} \left\{ \frac{h_2^*}{|h_2|} w_2 \right\} \right) \\ &= |h_2| \sqrt{(1-\alpha) P s_2} + \left(|h_2| \sqrt{\alpha P s_1} + n_2 \right) \end{aligned} \quad (5)$$

where n_1 and $n_2 \sim \mathcal{N}(0, N_0 / 2)$ are additive white Gaussian noise (AWGN). The notation $\mathcal{N}(\mu, \Sigma)$ denotes the normal distribution with mean μ and variance Σ . In the standard NOMA, the SIC is performed only on the user-1. Therefore, we define the perfect SIC as follows;

$$y_1 = r_1 - |h_1| \sqrt{(1-\alpha) P s_2} = |h_1| \sqrt{\alpha P s_1} + n_1. \quad (6)$$

In addition, if the performance of a superposition coding scheme is close to that of the perfect SIC, we use the word, the near-perfect SIC. We assume the 4PAM modulations for both users in the standard NOMA, i.e., the 4PAM/4PAM NOMA,

$$s_1, s_2 \in \left\{ +\frac{3}{\sqrt{5}}, +\frac{1}{\sqrt{5}}, -\frac{1}{\sqrt{5}}, -\frac{3}{\sqrt{5}} \right\}. \quad (7)$$

Let the symbol indexes for the user-1 and the

user-2 be $m_1, m_2 \in \{1, 2, 3, 4\}$.

III. Achieving Near-Perfect SIC Performance

The standard 4PAM NOMA with NSC achieves the perfect SIC performance for the power allocation factor range about $0\% \leq \alpha \leq 5\%$, and the trivial single power allocation factor $\alpha = 100\%$, i.e., no $|UI|^{[21]}$; 4PAM NSC is given by

$$\begin{aligned} s_1(m_1 = 4) &= +\frac{3}{\sqrt{5}}, & s_2(m_2 = 4) &= +\frac{3}{\sqrt{5}}, \\ s_1(m_1 = 3) &= +\frac{1}{\sqrt{5}}, & s_2(m_2 = 3) &= +\frac{1}{\sqrt{5}}, \\ s_1(m_1 = 2) &= -\frac{1}{\sqrt{5}}, & s_2(m_2 = 2) &= -\frac{1}{\sqrt{5}}, \\ s_1(m_1 = 1) &= -\frac{3}{\sqrt{5}}, & s_2(m_2 = 1) &= -\frac{3}{\sqrt{5}}. \end{aligned} \quad (8)$$

Note that the NSC has the performance crashes, at $\alpha = 0.1, 0.2, 4/13, 0.5, 9/13, 0.8, 0.9$, where the performance never improves, even though the total transmit power $P = \infty$. Then SSC fixes one performance crash at $\alpha = 0.1$, and adds one performance crash at $\alpha = 1^{[22]}$; 4PAM SSC of the user-1 is given by

$$\begin{aligned} \left\{ \begin{aligned} s_1(m_1 = 4 | m_2 = 4) &= +\frac{3}{\sqrt{5}} \\ s_1(m_1 = 3 | m_2 = 4) &= +\frac{1}{\sqrt{5}} \\ s_1(m_1 = 2 | m_2 = 4) &= -\frac{1}{\sqrt{5}} \\ s_1(m_1 = 1 | m_2 = 4) &= -\frac{3}{\sqrt{5}} \end{aligned} \right\} \left\{ \begin{aligned} s_1(m_1 = 4 | m_2 = 3) &= -\frac{3}{\sqrt{5}} \\ s_1(m_1 = 3 | m_2 = 3) &= -\frac{1}{\sqrt{5}} \\ s_1(m_1 = 2 | m_2 = 3) &= +\frac{1}{\sqrt{5}} \\ s_1(m_1 = 1 | m_2 = 3) &= +\frac{3}{\sqrt{5}} \end{aligned} \right\} \\ \left\{ \begin{aligned} s_1(m_1 = 4 | m_2 = 2) &= +\frac{3}{\sqrt{5}} \\ s_1(m_1 = 3 | m_2 = 2) &= +\frac{1}{\sqrt{5}} \\ s_1(m_1 = 2 | m_2 = 2) &= -\frac{1}{\sqrt{5}} \\ s_1(m_1 = 1 | m_2 = 2) &= -\frac{3}{\sqrt{5}} \end{aligned} \right\} \left\{ \begin{aligned} s_1(m_1 = 4 | m_2 = 1) &= -\frac{3}{\sqrt{5}} \\ s_1(m_1 = 3 | m_2 = 1) &= -\frac{1}{\sqrt{5}} \\ s_1(m_1 = 2 | m_2 = 1) &= +\frac{1}{\sqrt{5}} \\ s_1(m_1 = 1 | m_2 = 1) &= +\frac{3}{\sqrt{5}} \end{aligned} \right\}. \end{aligned} \quad (9)$$

Note that, for the user-2, the modulation signal mappings of SSC and CSC are exactly the same as that of NSC; only the mapping for the user-1 changes. The rest of the performance crashes are repaired by the CSC;

Table 1. Complement Superposition Coding

$\sqrt{5} \times s_1$	$\frac{m_2}{m_1}$	4				3				2				1			
		4	3	2	1	4	3	2	1	4	3	2	1	4	3	2	1
$4 / 29 < \alpha \leq 9 / 34$		+3	+1	-1	-3	-1	-3	+3	+1	+3	+1	-1	-3	-1	-3	+3	+1
$9 / 34 < \alpha \leq 9 / 25$		+3	+1	-1	-3	-1	+3	-3	+1	-3	+1	-1	+3	-1	-3	+3	+1
$9 / 25 < \alpha \leq 16 / 25$		+3	+1	-1	-3	-3	+3	+1	-1	-1	-3	+3	+1	+1	-1	-3	+3
$16 / 25 < \alpha \leq 0.8$		+3	+1	-1	-3	-3	+3	+1	-1	-3	+3	+1	-1	-1	-3	+3	+1
$0.8 < \alpha \leq 16 / 17$		+3	+1	-1	-3	+3	+1	-1	-3	-3	+3	+1	-1	-3	+3	+1	-1

• Key idea to design the CSC: to maximize the minimum of the minimum distances of pairwise signal points, for the given power allocation factor range.

For example, the value $4/29$ of α is calculated as follows.

$$\begin{aligned}
 \frac{3}{\sqrt{5}}\sqrt{1-\alpha} - \frac{2}{\sqrt{5}}\sqrt{\alpha} &= \frac{1}{\sqrt{5}}\sqrt{1-\alpha} + \frac{3}{\sqrt{5}}\sqrt{\alpha} \\
 3\sqrt{1-\alpha} - 2\sqrt{\alpha} &= \sqrt{1-\alpha} + 3\sqrt{\alpha} \\
 2\sqrt{1-\alpha} &= 5\sqrt{\alpha} \\
 4(1-\alpha) &= 25\alpha \\
 4 - 4\alpha &= 25\alpha \\
 4 &= 29\alpha \\
 \alpha &= \frac{4}{29} \simeq 0.138.
 \end{aligned} \tag{10}$$

Then 4PAM CSC of the user-1 is given in Table 1. For the performance calculation, we define the notation as

$$q^{(I;A)} = Q \left(\frac{|h_1| \sqrt{P} \left(\sqrt{(1-\alpha)} \frac{I}{\sqrt{5}} + \sqrt{\alpha} \frac{A}{\sqrt{5}} \right)}{\sqrt{N_0/2}} \right) \tag{11}$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$. Furthermore, to improve the readability, we define the compact notation as

$$\pm(I;A) \triangleq \pm q^{(I;A)}. \tag{12}$$

In addition, in order for the performance to be presented systematically, we define the conditional performance as follows

$$\mathcal{E}(m_1, m_2) \triangleq P_{e|h_1}^{(1; M=4; DSC; NOMA; optimal ML)}(m_1, m_2). \tag{13}$$

Thus, the resulting performance of DSC(NSC,SSC,CSC) is given by, for $0 \leq \alpha \leq 0.1$,

$$\begin{aligned}
 P_{e|h_1}^{(1; M=4; DSC(SSC); NOMA; optimal ML)} &\simeq \frac{1}{4} \times \frac{1}{4} \times 2 \times [\\
 &+ \mathcal{E}(m_1 = 4, m_2 = 4) + \mathcal{E}(m_1 = 4, m_2 = 3) \\
 &+ \mathcal{E}(m_1 = 3, m_2 = 4) + \mathcal{E}(m_1 = 3, m_2 = 3) \\
 &+ \mathcal{E}(m_1 = 2, m_2 = 4) + \mathcal{E}(m_1 = 2, m_2 = 3) \\
 &+ \mathcal{E}(m_1 = 1, m_2 = 4) + \mathcal{E}(m_1 = 1, m_2 = 3)]
 \end{aligned} \tag{14}$$

where

$$\begin{aligned}
 &+ \mathcal{E}(m_1 = 4, m_2 = 4) = \\
 &+ (+0; +1) - (+2; +5) + (+4; +1) - (+6; +5),
 \end{aligned} \tag{15}$$

$$\begin{aligned}
 &+ \mathcal{E}(m_1 = 4, m_2 = 3) = \\
 &- (+2; +5) + (+0; +1) + (+2; -5) - (+4; -1),
 \end{aligned} \tag{16}$$

$$\begin{aligned}
 &+ \mathcal{E}(m_1 = 3, m_2 = 4) = \\
 &+ (+0; +1) + (+0; +1) - (+2; +1) + (+2; +3) \\
 &- (+4; -1) + (+4; +1) - (+6; +1) + (+6; +3),
 \end{aligned} \tag{17}$$

$$\begin{aligned}
 &+ \mathcal{E}(m_1 = 3, m_2 = 3) = \\
 &+ (+2; +3) - (+2; +1) + (+0; +1) + (+0; +1) \\
 &- (+2; -3) + (+2; -1) - (+4; -1) + (+4; +1),
 \end{aligned} \tag{18}$$

$$\begin{aligned}
 &+ \mathcal{E}(m_1 = 2, m_2 = 4) = \\
 &+ (+0; +1) + (+0; +1) - (+2; -3) + (+2; -1) \\
 &- (+4; -1) + (+4; +1) - (+6; -3) + (+6; -1),
 \end{aligned} \tag{19}$$

$$\begin{aligned}
 &+ \mathcal{E}(m_1 = 2, m_2 = 3) = \\
 &+ (+2; -1) - (+2; -3) + (+0; +1) + (+0; +1) \\
 &- (+2; +1) + (+2; +3) - (+4; -1) + (+4; +1),
 \end{aligned} \tag{20}$$

$$\begin{aligned}
 &+ \mathcal{E}(m_1 = 1, m_2 = 4) = \\
 &+ (+0; +1) - (+2; -5) + (+4; -1) - (+6; -5),
 \end{aligned} \tag{21}$$

and

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 1, m_2 = 3) = \\
 & -(+2; -5) + (+0; +1) + (+2; +5) - (+4; +1), \tag{22}
 \end{aligned}$$

for $0.1 < \alpha \leq 4 / 29$,

$$\begin{aligned}
 P_{e|h}^{(1; M=4; DSC(SSC); NOMA; optimal ML)} & \simeq \frac{1}{4} \times \frac{1}{4} \times 2 \times [\\
 & +\mathcal{E}(m_1 = 4, m_2 = 4) + \mathcal{E}(m_1 = 4, m_2 = 3) \\
 & +\mathcal{E}(m_1 = 3, m_2 = 4) + \mathcal{E}(m_1 = 3, m_2 = 3) \\
 & +\mathcal{E}(m_1 = 2, m_2 = 4) + \mathcal{E}(m_1 = 2, m_2 = 3) \\
 & +\mathcal{E}(m_1 = 1, m_2 = 4) + \mathcal{E}(m_1 = 1, m_2 = 3)] \tag{23}
 \end{aligned}$$

where

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 4) = \\
 & +(+0; +1) - (+3; +2) + (+3; +4) - (+6; +5), \tag{24}
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 3) = \\
 & -(+2; +5) + (-1; +4) + (+1; -2) - (+4; -1), \tag{25}
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 3, m_2 = 4) = \\
 & +(+0; +1) + (+0; +1) - (+2; +1) + (+3; +0) \\
 & -(+3; +2) + (+4; +1) - (+6; +1) + (+6; +3), \tag{26}
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 3, m_2 = 3) = \\
 & +(+2; +3) - (+2; +1) + (+0; +1) + (+1; -2) \\
 & -(+1; -0) + (+2; -1) - (+4; -1) + (+4; +1), \tag{27}
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 4) = \\
 & +(+0; +1) + (+1; -2) - (+1; -0) + (+2; -1) \\
 & -(+4; -1) + (+5; -2) - (+5; -0) + (+6; -1), \tag{28}
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 3) = \\
 & +(+2; -1) - (+1; -0) + (+1; -2) + (+0; +1) \\
 & -(+2; +1) + (+3; +0) - (+3; +2) + (+4; +1), \tag{29}
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 1, m_2 = 4) = \\
 & +(-1; +4) + (+1; -2) - (+5; -4) + (+5; -2), \tag{30}
 \end{aligned}$$

and

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 1, m_2 = 3) = \\
 & +(+1; -2) + (-1; +4) - (+3; +2) + (+3; +4), \tag{31}
 \end{aligned}$$

for $4 / 29 < \alpha \leq 0.2$,

$$\begin{aligned}
 P_{e|h}^{(1; M=4; DSC(CSC); NOMA; optimal ML)} & \simeq \frac{1}{4} \times \frac{1}{4} \times 2 \times [\\
 & +\mathcal{E}(m_1 = 4, m_2 = 4) + \mathcal{E}(m_1 = 4, m_2 = 3) \\
 & +\mathcal{E}(m_1 = 4, m_2 = 2) + \mathcal{E}(m_1 = 4, m_2 = 1) \\
 & +\mathcal{E}(m_1 = 2, m_2 = 4) + \mathcal{E}(m_1 = 2, m_2 = 3) \\
 & +\mathcal{E}(m_1 = 2, m_2 = 2) + \mathcal{E}(m_1 = 2, m_2 = 1)] \tag{32}
 \end{aligned}$$

where

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 4) = +(+0; +1) \\
 & -(+2; +3) + (+3; +3) - (+6; +3) + (+6; +5), \tag{33}
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 3) = -(+2; +3) \\
 & +(+0; +1) + (+1; -1) - (+4; -1) + (+4; +1), \tag{34}
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 2) = -(+4; -1) \\
 & +(+2; -3) + (-1; +3) - (+2; +3) + (+2; +5), \tag{35}
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 1) = -(+6; +3) \\
 & +(+4; +1) - (+3; +1) + (+0; +1) + (+0; +1), \tag{36}
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 4) = \\
 & +(+0; +1) + (+1; -1) - (+4; -1) + (+5; -1), \tag{37}
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 3) = \\
 & +(+2; -3) + (-1; +3) - (+2; +3) + (+3; +3), \tag{38}
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 2) = \\
 & +(+4; +1) - (+3; +1) + (+0; +1) + (+1; -1), \tag{39}
 \end{aligned}$$

and

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 1) = \\
 & +(+6; +3) - (+5; -3) + (+2; -3) + (-1; +3), \tag{40}
 \end{aligned}$$

for $0.2 < \alpha \leq 9 / 34$,

$$\begin{aligned}
 P_{e|h}^{(1; M=4; DSC(CSC); NOMA; optimal ML)} & \simeq \frac{1}{4} \times \frac{1}{4} \times 2 \times [\\
 & +\mathcal{E}(m_1 = 4, m_2 = 4) + \mathcal{E}(m_1 = 4, m_2 = 3) \\
 & +\mathcal{E}(m_1 = 4, m_2 = 2) + \mathcal{E}(m_1 = 4, m_2 = 1) \\
 & +\mathcal{E}(m_1 = 2, m_2 = 4) + \mathcal{E}(m_1 = 2, m_2 = 3) \\
 & +\mathcal{E}(m_1 = 2, m_2 = 2) + \mathcal{E}(m_1 = 2, m_2 = 1)] \tag{41}
 \end{aligned}$$

where

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 4) = +(+0; +1) \\
 & -(+2; +3) + (+3; +3) - (+5; +5) + (+6; +5), \tag{42}
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 3) = -(+2; +3) \\
 & +(+0; +1) + (+1; -1) - (+3; +1) + (+4; +1), \quad (43)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 2) = -(+4; -1) \\
 & +(+2; -3) + (-1; +3) - (+2; +3) + (+2; +5), \quad (44)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 1) = -(+6; +3) \\
 & +(+4; +1) - (+3; +1) + (+1; -1) + (+0; +1), \quad (45)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 4) = \\
 & +(-1; +3) + (+1; -1) - (+4; -1) + (+5; -1) \quad (46)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 3) = \\
 & +(+1; -1) + (-1; +3) - (+2; +3) + (+3; +3), \quad (47)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 2) = \\
 & +(+3; +3) - (+3; +1) + (+0; +1) + (+1; -1), \quad (48)
 \end{aligned}$$

and

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 1) = \\
 & +(+5; -1) - (+5; +3) + (+2; -3) + (-1; +3), \quad (49)
 \end{aligned}$$

for $9 / 34 < \alpha \leq 4 / 13$,

$$\begin{aligned}
 & P_{e|h_1}^{(1; M=4; DSC(CSC); NOMA; optimal ML)} \simeq \frac{1}{4} \times \frac{1}{4} \times 2 \times [\\
 & +\mathcal{E}(m_1 = 4, m_2 = 4) + \mathcal{E}(m_1 = 4, m_2 = 3) \\
 & +\mathcal{E}(m_1 = 4, m_2 = 2) + \mathcal{E}(m_1 = 4, m_2 = 1) \quad (50)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 4) + \mathcal{E}(m_1 = 2, m_2 = 3) \\
 & +\mathcal{E}(m_1 = 2, m_2 = 2) + \mathcal{E}(m_1 = 2, m_2 = 1)]
 \end{aligned}$$

where

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 4) = +(+0; +1) \\
 & -(+3; +2) + (+3; +3) - (+5; +4) + (+6; +5), \quad (51)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 3) = -(+2; +3) \\
 & +(-1; +2) + (+1; -1) - (+3; +0) + (+4; +1), \quad (52)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 2) = -(+4; +5) \\
 & +(+1; +4) - (+1; +3) + (-1; +2) + (+2; -1), \quad (53)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 1) = -(+6; +3) \\
 & +(+3; +2) - (+3; +1) + (+1; -0) + (+0; +1), \quad (54)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 4) = \\
 & +(-1; +2) + (+1; -1) - (+3; -0) + (+5; -1), \quad (55)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 3) = \\
 & +(+1; +4) - (+1; +3) + (-1; +2) + (+3; -3), \quad (56)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 2) = \\
 & +(+3; +2) - (+3; +1) + (+1; +0) + (+1; -1), \quad (57)
 \end{aligned}$$

and

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 1) = \\
 & +(+5; -2) - (+5; -3) + (+3; -4) + (-1; +3), \quad (58)
 \end{aligned}$$

for $4 / 13 < \alpha \leq 9 / 25$,

$$\begin{aligned}
 & P_{e|h_1}^{(1; M=4; DSC(CSC); NOMA; optimal ML)} \simeq \frac{1}{4} \times \frac{1}{4} \times 2 \times [\\
 & +\mathcal{E}(m_1 = 4, m_2 = 4) + \mathcal{E}(m_1 = 4, m_2 = 3) \\
 & +\mathcal{E}(m_1 = 4, m_2 = 2) + \mathcal{E}(m_1 = 4, m_2 = 1) \quad (59)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 4) + \mathcal{E}(m_1 = 2, m_2 = 3) \\
 & +\mathcal{E}(m_1 = 2, m_2 = 2) + \mathcal{E}(m_1 = 2, m_2 = 1)]
 \end{aligned}$$

where

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 4) = +(+0; +1) \\
 & -(+2; +5) + (+3; +3) - (+5; +4) + (+6; +5), \quad (60)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 3) = -(+2; +3) \\
 & +(+1; -1) + (+1; -1) - (+3; +0) + (+4; +1), \quad (61)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 2) = -(+4; +5) \\
 & +(+3; +1) - (+1; +3) + (-1; +2) + (+2; -1), \quad (62)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 4, m_2 = 1) = -(+6; +3) \\
 & +(+5; -2) - (+3; +1) + (+1; -0) + (+0; +1), \quad (63)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 4) = \\
 & +(-1; +2) + (+1; -1) - (+5; -3) + (+5; -1), \quad (64)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 3) = \\
 & +(+1; +4) - (+1; +3) + (-3; +5) + (+3; -3), \quad (65)
 \end{aligned}$$

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 2) = \\
 & +(+3; +2) - (+3; +1) + (-1; +3) + (+1; -1), \quad (66)
 \end{aligned}$$

and

$$\begin{aligned}
 & +\mathcal{E}(m_1 = 2, m_2 = 1) = \\
 & +(+5; -2) - (+5; -3) + (+1; -1) + (-1; +3), \quad (67)
 \end{aligned}$$

for $9 / 25 < \alpha \leq 0.5$,

$$P_{e|h}^{(1; M=4; DSC(CSC); NOMA; optimal ML)} \simeq \frac{1}{4} \times \frac{1}{4} \times 2 \times [\begin{aligned} & +\mathcal{E}(m_1 = 4, m_2 = 3) = \\ & +(+0;+1) + (+1;+0) - (+3;+4), \end{aligned} \tag{79}$$

$$+\mathcal{E}(m_1 = 4, m_2 = 4) + \mathcal{E}(m_1 = 4, m_2 = 3) \tag{68}$$

$$+\mathcal{E}(m_1 = 4, m_2 = 2) + \mathcal{E}(m_1 = 4, m_2 = 1) \tag{80}$$

$$+\mathcal{E}(m_1 = 3, m_2 = 4) + \mathcal{E}(m_1 = 3, m_2 = 3) \tag{81}$$

$$+\mathcal{E}(m_1 = 1, m_2 = 4) + \mathcal{E}(m_1 = 1, m_2 = 3)]$$

where

$$+\mathcal{E}(m_1 = 4, m_2 = 4) = \tag{69}$$

$$+(-1;+2) + (+2;-1) - (+6;+1),$$

$$+\mathcal{E}(m_1 = 4, m_2 = 3) = \tag{70}$$

$$+(+1;+0) + (+0;+1) - (+4;+3),$$

$$+\mathcal{E}(m_1 = 4, m_2 = 2) = \tag{71}$$

$$+(+3;-2) + (-2;+3) - (+2;+5),$$

$$+\mathcal{E}(m_1 = 4, m_2 = 1) = \tag{72}$$

$$+(+5;+4) - (+4;+3) + (+0;+1),$$

$$+\mathcal{E}(m_1 = 3, m_2 = 4) = \tag{73}$$

$$+(-0;+1) + (+1;-0) - (+5;+2) + (+6;+3),$$

$$+\mathcal{E}(m_1 = 3, m_2 = 3) = \tag{74}$$

$$+(+2;-1) + (-1;+2) - (+3;+4) + (+4;+5),$$

$$+\mathcal{E}(m_1 = 1, m_2 = 4) = +(-2;+3) + (+4;-3), \tag{75}$$

and

$$+\mathcal{E}(m_1 = 1, m_2 = 3) = +(+0;+1) + (+2;-1), \tag{76}$$

for $0.5 < \alpha \leq 16 / 25$,

$$P_{e|h}^{(1; M=4; DSC(CSC); NOMA; optimal ML)} \simeq \frac{1}{4} \times \frac{1}{4} \times 2 \times [\begin{aligned} & +\mathcal{E}(m_1 = 4, m_2 = 4) + \mathcal{E}(m_1 = 4, m_2 = 3) \\ & +\mathcal{E}(m_1 = 4, m_2 = 2) + \mathcal{E}(m_1 = 4, m_2 = 1) \end{aligned} \tag{77}$$

$$+\mathcal{E}(m_1 = 3, m_2 = 4) + \mathcal{E}(m_1 = 3, m_2 = 3)$$

$$+\mathcal{E}(m_1 = 1, m_2 = 4) + \mathcal{E}(m_1 = 1, m_2 = 3)]$$

where

$$+\mathcal{E}(m_1 = 4, m_2 = 4) = \tag{78}$$

$$+(-2;+3) + (+3;-2) - (+5;+2),$$

$$+\mathcal{E}(m_1 = 3, m_2 = 4) = \tag{82}$$

$$+(-1;+2) + (+2;-1) - (+4;+3) + (+5;+4),$$

$$+\mathcal{E}(m_1 = 3, m_2 = 3) = \tag{83}$$

$$+(+1;+0) + (+0;+1) - (+2;+5) + (+3;+6),$$

$$+\mathcal{E}(m_1 = 1, m_2 = 4) = +(+3;-2) + (-3;+4), \tag{84}$$

and

$$+\mathcal{E}(m_1 = 1, m_2 = 3) = +(+1;+0) + (-1;+2), \tag{85}$$

for $16 / 25 < \alpha \leq 0.8$,

$$P_{e|h}^{(1; M=4; DSC(CSC); NOMA; optimal ML)} \simeq \frac{1}{4} \times \frac{1}{4} \times 2 \times [\begin{aligned} & +\mathcal{E}(m_1 = 4, m_2 = 4) + \mathcal{E}(m_1 = 4, m_2 = 3) \\ & +\mathcal{E}(m_1 = 4, m_2 = 2) + \mathcal{E}(m_1 = 4, m_2 = 1) \end{aligned} \tag{86}$$

$$+\frac{1}{4} \times \frac{1}{4} \times 4 \times [\begin{aligned} & +\mathcal{E}(m_1 = 2, m_2 = 4) + \mathcal{E}(m_1 = 2, m_2 = 3) \end{aligned}]$$

where

$$+\mathcal{E}(m_1 = 4, m_2 = 4) = \tag{87}$$

$$+(-1;+2) + (+3;-1) - (+5;+4),$$

$$+\mathcal{E}(m_1 = 4, m_2 = 3) = \tag{88}$$

$$+(+1;+0) + (+1;+1) - (+3;+6),$$

$$+\mathcal{E}(m_1 = 4, m_2 = 2) = \tag{89}$$

$$- (+1;+6) + (+3;-0) + (-1;+1),$$

$$+\mathcal{E}(m_1 = 4, m_2 = 1) = \tag{90}$$

$$+(+5;+6) - (+3;+5) + (+1;+0),$$

$$+\mathcal{E}(m_1 = 2, m_2 = 4) = +(-3;+3) + (+3;-1), \tag{91}$$

and

$$+\mathcal{E}(m_1 = 2, m_2 = 3) = +(-1;+1) + (+1;+1), \tag{92}$$

for $0.8 < \alpha \leq 16 / 17$,

$$\begin{aligned}
 P_{e|h_1}^{(1; M=4; DSC(CSC); NOMA; optimal ML)} &\simeq \frac{1}{4} \times \frac{1}{4} \times 2 \times [\\
 +\mathcal{E}(m_1 = 4, m_2 = 4) + \mathcal{E}(m_1 = 4, m_2 = 3) & \\
 +\frac{1}{4} \times \frac{1}{4} \times 2 \times 3 \times [& \\
 +\mathcal{E}(m_1 = 3, m_2 = 4) + \mathcal{E}(m_1 = 3, m_2 = 3) &]]
 \end{aligned} \tag{93}$$

where

$$+\mathcal{E}(m_1 = 4, m_2 = 4) = +(+3; +0) + (+3; +6), \tag{94}$$

$$+\mathcal{E}(m_1 = 4, m_2 = 3) = +(+1; +0) + (+1; +6), \tag{95}$$

$$+\mathcal{E}(m_1 = 3, m_2 = 4) = +(-3; +2) + (+3; +0), \tag{96}$$

and

$$+\mathcal{E}(m_1 = 3, m_2 = 3) = +(-1; +2) + (+1; +0), \tag{97}$$

for $16/17 < \alpha \leq 1$,

$$\begin{aligned}
 P_{e|h_1}^{(1; M=4; DSC(NSC); NOMA; optimal ML)} &= \frac{1}{4} \times \frac{1}{4} \times 6 \times [\\
 +\mathcal{E}(m_1 = 4, m_2 = 4) + \mathcal{E}(m_1 = 4, m_2 = 3) & \\
 +\mathcal{E}(m_1 = 4, m_2 = 2) + \mathcal{E}(m_1 = 4, m_2 = 1) &]
 \end{aligned} \tag{98}$$

where

$$+\mathcal{E}(m_1 = 4, m_2 = 4) = +(+3; +1), \tag{99}$$

$$+\mathcal{E}(m_1 = 4, m_2 = 3) = +(+1; +1), \tag{100}$$

$$+\mathcal{E}(m_1 = 4, m_2 = 2) = +(-1; +1), \tag{101}$$

and

$$+\mathcal{E}(m_1 = 4, m_2 = 1) = +(-3; +1). \tag{102}$$

Then the fading performance is calculated by

$$\begin{aligned}
 P_e^{(1; M=4; DSC; NOMA; optimal ML)} & \\
 = \mathbb{E}_{h_1} \left[P_{e|h_1}^{(1; M=4; DSC; NOMA; optimal ML)} \right]. & \tag{103}
 \end{aligned}$$

Thus we can use the well-known Rayleigh fading integration formula, with $\gamma_b = \Sigma_1 P \left(\sqrt{(1-\alpha)} \frac{I}{\sqrt{5}} + \sqrt{\alpha} \frac{A}{\sqrt{5}} \right)^2 / N_0$,

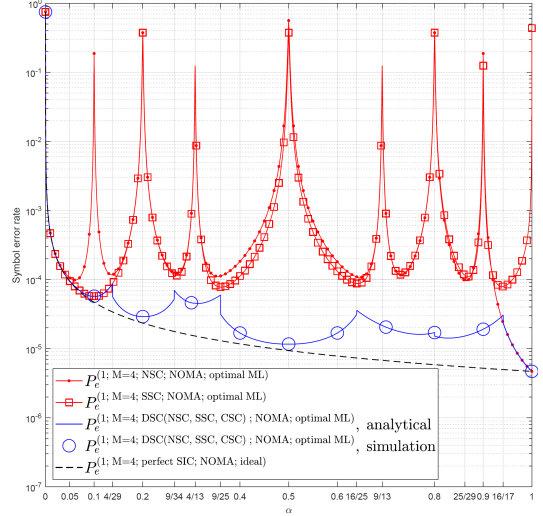


Fig. 1. Comparison of symbol error rates for 4PAM NOMA with DSC, SSC, NSC, and perfect SIC, for the user-1 only ($0 < \alpha < 1$).

$$\int_0^\infty q(I;A) \frac{1}{\gamma_b} e^{-\frac{\gamma}{\gamma_b}} d\gamma = \frac{1}{2} \left(1 - \sqrt{\frac{\gamma_b}{1 + \gamma_b}} \right). \tag{104}$$

IV. Results and Discussions

We assume the perfect channel state information (CSI) is available at the receiver. Therefore, the performance degradation due to the CSI errors can be observed in the practical systems. Assume that

the channel gain variance $\Sigma_1 = (2.0)^2$ and $P / N_0 = 50$ dB. In Fig. 1, we compare

$P_e^{(1; M=4; DSC; NOMA; optimal ML)}$ to $P_e^{(1; M=4; NSC; NOMA; optimal ML)}$ in [21] and $P_e^{(1; M=4; SSC; NOMA; optimal ML)}$ in [22] and

$P_e^{(1; M=4; NOMA; ideal perfect SIC)}$. As shown in Fig. 1, NOMA with NSC has the performance crashes at $\alpha \simeq 0.1, 0.2, 0.3, 0.5, 0.7, 0.8, 0.9$ and the performance of NOMA with SSC crashes at $\alpha \simeq 0.2, 0.3, 0.5, 0.7, 0.8, 0.9, 1$. However, DSC NOMA fixes all the performance crashes and effectively achieves the near-perfect SIC performance. We also show simulation results in Fig. 1, which are in good agreement with analytical

results.

One important comment is that for all the superposition coding schemes, the performance of the user-2 is the same exactly, because the signal mapping for the user-2 is not changed. Therefore, the performance crashes for the user-2 in [20] are not fixed and remain for all the superposition coding schemes.

V. Conclusion

In this paper, we proposed CSC, and with DSC techniques, it was shown analytically that the near-perfect SIC performance can be obtained for the multilevel modulation, just as for the single level modulation. Specifically, the seven performance crashes in NSC and SSC for 4PAM NOMA were fixed. In result, the near-perfect SIC performance could be implemented for the multilevel modulation.

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