# Static Power Control for D2D Communication Underlaid Cellular Downlink Networks 

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#### Abstract

In device-to-device (D2D) communication underlaid cellular networks, a problem of balancing transmit power of a base station and D2D transmitters is important to effectively control mutual interference. To address this, we obtain analytical expressions for the ergodic spectral efficiencies by leveraging tools of stochastic geometry. With the derived expressions, an optimization problem is formulated to find the optimal power ratio between the D2D transmitter and the base station to maximize the sum spectral efficiencies. The simulation shows that using the optimal power ratio increases the sum spectral efficiencies. A benefit of the proposed optimal power ratio is that it does not have to be changed frequently.


Key Words : Power control, Stochastic geometry, D2D underlaid cellular networks

## I . Introduction

Device-to-device (D2D) communication enables low latency proximity-based services. When underlaying a cellular network, D2D communication also can increase the area spectral efficiency due to spatial reuse gains through local D2D traffic. Despite such potential benefits, sharing spectrum between D2D and cellular communication is challenging because it introduces additional interference sources. For example, in a D2D underlaid cellular network, cellular users experience not only intra-tier interference from other cells, but also cross-tier interference from D2D transmitters.

As a result, interference management is important for successful spectrum sharing between cellular and D2D links ${ }^{[1,2]}$. Especially, to manage the interference with small amount of associated overheads, static power control is a simple but effective approach. In this letter, we consider a static power control problem for D2D underlaid cellular downlink networks. The man objective of the problem is to find the optimal power balance between the D2D links and the cellular base station. To address the problem, we first derive analytical expressions for the ergodic spectral efficiencies of D2D and cellular links relying on a spatial random network model built upon stochastic geometry. Then, we formulate an optimization problem for finding the optimal power ratio between the D2D transmit power and the cellular transmit power to maximize the sum spectral efficiencies. Via simulation, we demonstrate that the proposed static power control solution provides the maximum sum spectral efficiencies. A main merit of the proposed solution is small amount of overheads. Applying the proposed solution, BSs and D2D transmitters use fixed power level determined by the ratio of their densities, so that the transmit power does not have to be changed frequently.

## II. System Model

In this section, we introduce the system model used in this letter. We consider a stochastic network model for modeling a D2D underlaid downlink cellular network, where each D2D link and cellular link share the same spectrum. Specifically, the locations of the BSs equipped with a single-antenna are distributed according to a homogeneous Poisson point process (PPP), denoted by $\Phi_{b s}$ with density $\lambda_{b s}$. Each mobile user is also distributed as a homogeneous PPP with density $\lambda_{\text {user }}$, and associated with the closest BS yielding the minimum average path-loss.

[^0]The locations of the D2D transmitters are also distributed in the network plane according to a homogeneous PPP denoted by $\Phi_{d 2 d}$ with density $\lambda_{d 2 d}$. We assume that $\Phi_{d 2 d}$ is independent to $\Phi_{b s}$. Each D2D receiver is located within a certain annulus centered at the paired D2D transmitter location. Specifically, we assume that the D2D receiver located at $\tilde{z}_{k}$ communicates with the D2D transmitter whose the location is $z_{k}$, where $\tilde{z}_{k}$ is uniformly distributed in the annulus centered at $z_{k}$ with inner radius 1 and outer radius $R_{d}$. Here, the radius $\quad R_{d}$ implies the maximum D2D communication range. By this construction, a D2D receiver does not necessarily communicate with the nearest D2D transmitter.

Now we define the small-scale fading power of each link. $H_{i, j}$ refers to the small-scale fading power between the j-th BS and the i-th mobile user (cellular tier), while $\widetilde{H}_{k, l}$ is the small-scale fading power between the 1-th BS and the k-th D2D receiver (cross tier). $G_{i, j}$ means the small-scale fading power between the j -th D2D transmitter and the i-th D2D receiver (D2D tier), while $\tilde{G}_{k, l}$ denotes the small-scale fading power between the 1-th D2D transmitter and the k-th mobile user (cross tier). All the fading power coefficients are exponentially distributed with unit mean. The BS and the D2D transmitter use the transmit power $P_{b s}$ and $P_{d 2 d}$.

Per Slivnyak's theorem ${ }^{[2]}$, let us consider the typical mobile user 1 located at the origin and served by the BS located at $d_{1}$. The instantaneous signal-to-interference ratio (SIR) of the typical mobile user is defined as

$$
\begin{equation*}
S I R_{1}^{m}=\frac{H_{1,1} P_{b s}\left\|d_{1}\right\|^{-\beta}}{I_{m, b s}+I_{m, d 2 d}} \tag{1}
\end{equation*}
$$

where $I_{m, b s}$ and $I_{m, d 2 d}$ denote the aggregated intra-tier (from other BSs) and cross-tier (from D2D transmitters) interference, respectively, and $\beta$ means the path-loss exponent. Throughout this letter, we assume $\beta=4$. Similar to (1), the instantaneous SIR
of the k-th D2D receiver is given by

$$
\begin{equation*}
\operatorname{SIR}_{k}^{d 2 d}=\frac{G_{k, k} P_{d 2 d}\left\|z_{k}-\tilde{z}_{k}\right\|^{-\beta}}{I_{d 2 d, b s}+I_{d 2 d, d 2 d}} \tag{2}
\end{equation*}
$$

where $I_{d 2 d, b s}$ and $I_{d 2 d, d 2 d}$ denote the aggregated cross-tier (from BSs) and intra-tier (from other D2D transmitters) interference, respectively. Note that we only consider the interference and ignore the noise. This is because the interference is a dominant factor to determine the achievable rate performance in dense cellular and D2D networks ${ }^{[3]}$.

## III. Performance Analysis

In this section, we characterize the ergodic spectral efficiencies as functions of the major system parameters. The derived expressions are further exploited to find the optimal power ratio.

We first obtain the ergodic spectral efficiency of the cellular user, denoted by $R_{b s}$, as follows.

$$
\begin{equation*}
\log _{2(e)} \int_{0}^{\infty} \frac{1 /(1+z)}{1+A(z, \beta)+\frac{\lambda_{d 2 d}}{\lambda_{b s}}\left(\frac{P_{d 2 d}}{P_{b s}}\right)^{2 / \beta} \frac{z^{2 / \beta}}{\operatorname{sinc}(2 / \beta)}} \mathrm{d} z \tag{3}
\end{equation*}
$$

where $\quad A(a, b)=\frac{2 a}{b-2}{ }_{2} \mathrm{~F}_{1}(1,1-2 / b, 2-2 / b,-a)$ with ${ }_{2} \mathrm{~F}_{1}$ is the Gauss hypergeometric function. Due to space limitation, we provide the proof sketch of (3). The ergodic spectral efficiency of the cellular user is defined as

$$
\begin{equation*}
\frac{1}{\ln 2} E\left[\ln \left(1+S I R_{1}^{m}\right)\right] \tag{4}
\end{equation*}
$$

By leveraging ${ }^{[3]}$, we represent (4) as

$$
\begin{equation*}
\int_{0}^{\infty} \frac{1}{1+z} L_{I_{m, b s}}\left(z\left\|d_{1}\right\|^{\beta}\right) L_{I_{m, d d s}}\left(z\left\|d_{1}\right\|^{\beta} \frac{P_{d 2 d}}{P_{b s}}\right) \mathrm{d} z \tag{5}
\end{equation*}
$$

where $L_{I_{m, b s}}$ and $L_{I_{m, d 2 d}}$ are the Laplace transform of each interference term. By using the tools of stochastic geometry, each Laplace transform is computed as in the following forms.

$$
\begin{gather*}
L_{I_{m b s}}\left(s \mid\left\|d_{1}\right\|=D\right)=\exp \left(-2 \pi \lambda_{b s} \int_{D}^{\infty} \frac{u}{1+s^{-1} u^{\beta}} \mathrm{d} u\right)  \tag{7}\\
L_{I_{m, d 2 d}}(s)=\exp \left(-\frac{\lambda_{d 2 d} \pi s^{2 / \beta}}{\operatorname{sinc}(2 / \beta)}\right) \tag{8}
\end{gather*}
$$

After inserting (7) and (8) into (5), we marginalize the expression regarding $D$ by using the first-touch distribution ${ }^{[4]}$. This completes the proof.

Subsequently, we derive the ergodic spectral efficiency of the D2D receiver, denoted as $R_{d 2 d}$, as in the following form.

$$
\begin{equation*}
\frac{1}{\ln 2} \int_{0}^{\infty} \frac{e^{-\pi \lambda_{d 2 d} C(z, \beta, \bar{\lambda}, \bar{P})}-e^{\lambda_{d 2 d} \pi R_{d}^{2} C(z, \beta, \bar{\lambda}, \bar{P})}}{(1+z) \lambda_{d 2 d} \pi R_{d}^{2} C(z, \beta, \bar{\lambda}, \bar{P})} \mathrm{d} z \tag{8}
\end{equation*}
$$

where

$$
\begin{equation*}
C(z, \beta, \bar{\lambda}, \bar{P})=\frac{z^{2 / \beta}}{\sin c(2 / \beta)}\left\{\frac{\lambda_{\mathrm{bs}}}{\lambda_{\mathrm{d} 2 \mathrm{~d}}}\left(\frac{\mathrm{P}_{\mathrm{bs}}}{\mathrm{P}_{\mathrm{d} 2 \mathrm{~d}}}\right)^{2 / \beta}+1\right\} \tag{9}
\end{equation*}
$$

We omit the proof since it is similar to that of the cellular user case.


Fig. 1. Average cell spectral efficiencies as a function of power ratio when $R_{d}=50, \lambda_{b s}=0.00055 / \pi$

## IV. Static Power Control

By leveraging the obtained analytical results, we formulate the power control optimization problem as follows.

$$
\begin{equation*}
\left(\frac{P_{d 2 d}}{P_{b s}}\right)^{\star}=\arg \max \left(R_{b s}+\frac{\lambda_{d 2 d}}{\lambda_{b s}} R_{d 2 d}\right) \tag{10}
\end{equation*}
$$

Note that $1 / \lambda_{b s}$ is the averaged volume of a cell and $\lambda_{d 2 d} / \lambda_{b s}$ denotes the average number of D2D links per a cell. Therefore, the objective function in (10) implies the sum spectral efficiencies of all active communicating links per cell in a spatial average sense. We depict the simulation results of the cell spectral efficiencies for various power ratio in Fig.1. As observed in the figure, the optimal power ratio maximizes the sum spectral efficiencies by striking a balance between the BS transmit power and the D 2 D transmit power. In addition to that, the optimal power ratio decreases as the density ratio increases (the D2D density increases). This makes sense since when the number of the underlaid D2D links increases, the interference on the cellular user becomes severe. We note that since the optimum power ratio is only determined by the ratio of the BS and D2D densities, the proposed solution is changed in a slow manner; resulting in few amount of the associated overheads.

## V. Conclusions

In this letter, we derived analytical expressions for the ergodic spectral efficiencies of D2D and cellular links, where the D2D links are underlaid in a cellular downlink network. Using this expressions, we formulated the optimization problem of finding the optimal power ratio to maximize the sum spectral efficiencies. The optimal power ratio keeps a balance of the D2D and cellular BS transmit power, which improves the sum spectral efficiencies. The proposed static power control solution is attractive in terms of the small amount of the overheads.

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    논문번호 : 202001-010-A-LU, Received January 21, 2020; Revised February 18, 2020; Accepted February 19, 2020

