Orthogonal Dependent Raise for 4-Ary Transmission NOMA

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ABSTRACT

For 4-ary pulse amplitude modulation (PAM) non-orthogonal multiple access (NOMA), we present the ORthogonal DEpendent Raise (ORDER), which is the correlated superposition coding (SC). It is shown that the ORDER NOMA not only achieves the perfect successive interference cancellation (SIC) symbol error-rate (SER) performance, but also repairs the seven SER performance degradations in the standard 4PAM NOMA. In result, the ORDER NOMA could be a promising SC for the 4-ary transmission NOMA.

Key Words : NOMA, superposition coding, successive interference cancellation, power allocation, correlation coefficient.

Ⅰ. Introduction

Recently non-orthogonal multiple access (NOMA) has received significant attentions in the fifth generation (5G) mobile communication [1-6]. In NOMA, successive interference cancellation (SIC) is performed at the stronger channel user, to remove the interference due to superposition coding (SC). We present the ORthogonal DEpendent Raise (ORDER), which is the correlated SC for 4-ary pulse amplitude modulation (4PAM) NOMA.

- Our contributions: 1. We propose the correlated SC for the 4-ary transmission NOMA. 2. We achieve the perfect SIC symbol error-rate (SER) performance for the stronger channel user. 3. We repair the 7 performance degradations for the weaker channel user in the standard 4PAM NOMA.

Ⅱ. System and Channel Model

Assume that the constant total transmitted power is P, and the power of the user-1 signal s_1 and the user-2 signal s_2 is normalized as $\mathbb{E}\left[|s_1|^2\right]=\mathbb{E}\left[|s_2|^2\right]=1$, with the correlation coefficient $\rho=\mathbb{E}\left[s_1 s_2^*\right]$. The power allocation factor is α with $0 \le \alpha \le 1$. Then the superimposed signal is expressed by

$$x=\sqrt{\alpha P_A}s_1+\sqrt{(1-\alpha)P_A}s_2. \tag{1}$$

where the total allocated power is given by

$$P_A=\frac{P}{1+2\mathrm{Re}\{\rho\}\sqrt{\alpha}\sqrt{1-\alpha}}. \tag{2}$$

Before the SIC is performed on the user-1 with the better channel condition, the received signals of the user-1 and user-2 are represented as

$$\begin{aligned} r_1&=|h_1|\sqrt{\alpha P_A}s_1+\left(|h_1|\sqrt{(1-\alpha)P_A}s_2+n_1\right)\\ r_2&=|h_2|\sqrt{(1-\alpha)P_A}s_2+\left(|h_2|\sqrt{\alpha P_A}s_1+n_2\right)\end{aligned} \tag{3}$$

where h_1 and h_2 are the channel gains, with $|h_1|>|h_2|$, n_1 and $n_2 \sim \mathcal{N}(0,N_0/2)$ are additive white Gaussian noise (AWGN), and N_0 is one-sided power spectral density. The notation $\mathcal{N}(\mu,\Sigma)$ denotes the normal distribution with mean μ and variance Σ.

In addition, we should mention the practical issues on 4PAM; in practical communication systems, quadrature phase shift keying (QPSK) is used frequently. However, in the academic perspective, if we can calculate the performance of BPSK, of which the quadrature version is QPSK, we also obtain that of QPSK, i.e., the performance of BPSK with the half of power of QPSK (assuming Gray mapping). In the similar way, we can think 4PAM and 16-ary quadrature amplitude modulation (QAM).

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논문번호 : 202003-045-A-LU, Received March 3, 2020; Revised March 31, 2020; Accepted April 6, 2020

Ⅲ. Orthogonal Dependent Raise

First, we start the independent SC in the standard 4PAM NOMA, and then design the correlated SC. Let the message indexes for the user-1 and the user-2 be $m_1, m_2 \in \{4,3,2,1\}$, with

$$s_1, s_2 \in \left\{+\frac{3}{\sqrt{5}}, +\frac{1}{\sqrt{5}}, -\frac{1}{\sqrt{5}}, -\frac{3}{\sqrt{5}}\right\}. \tag{4}$$

Then the independently superimposed signal in the standard 4PAM NOMA can be represented as

$$x_s(m_1, m_2) = (2m_1 - 5)\sqrt{\frac{\alpha P_A}{5}} + (2m_2 - 5)\sqrt{\frac{(1-\alpha)P_A}{5}}. \tag{5}$$

Note that the order of the 16 superimposed signals changes, dependent on the power allocation.

Now, we "orthogonalize and raise" the user-2 signal, in the power domain, "dependent" on the standard 4PAM user-1 signal, i.e., the ORDER. Thus, the ORDER is represented by

$$\begin{aligned} x_c(m_1 = 4, m_2) &= +3\sqrt{\frac{\alpha P_A}{5}} + (-m_2 + 7)\sqrt{\frac{2(1-\alpha)P_A}{25}} \\ x_c(m_1 = 3, m_2) &= +\sqrt{\frac{\alpha P_A}{5}} + (m_2 - 1)\sqrt{\frac{2(1-\alpha)P_A}{25}} \\ x_c(m_1 = 2, m_2) &= -\sqrt{\frac{\alpha P_A}{5}} - (m_2 - 1)\sqrt{\frac{2(1-\alpha)P_A}{25}} \\ x_c(m_1 = 1, m_2) &= -3\sqrt{\frac{\alpha P_A}{5}} - (-m_2 + 7)\sqrt{\frac{2(1-\alpha)P_A}{25}} \end{aligned} \tag{6}$$

Note that in the ORDER NOMA, the order of the 16 superimposed signals does not change, i.e., independent from the power allocation. The motivation and key idea is as follows. For the standard NOMA, for example, the amplitude of the independently superimposed signal is reduced

$$x_s(m_1 = 4, m_2 = 2) = +3\sqrt{\frac{\alpha P_A}{5}} - \sqrt{\frac{(1-\alpha)P_A}{5}}. \tag{7}$$

Therefore, in order to avoid the reduction of the amplitude, the correlated SC is designed to "interfere constructively"

$$x_c(m_1 = 4, m_2 = 2) = +3\sqrt{\frac{\alpha P_A}{5}} + 5\sqrt{\frac{2(1-\alpha)P_A}{25}}. \tag{8}$$

In this sense, in the power domain, the user-2 signal is orthogonalized; in this case, it looks like one signal is raised (stacked) on the top of another.

Ⅳ. SER Derivation

The correlation coefficient of the correlated SC in the ORDER NOMA is calculated by

$$\rho = \frac{3}{\sqrt{10}} \simeq 0.9487. \tag{9}$$

Consider Rayleigh fading channels with $\mathbb{E}\left[|h_1|^2\right] = \Sigma_1$ and $\mathbb{E}\left[|h_2|^2\right] = \Sigma_2$. For the simplification of Rayleigh fading SER performance, we define the notation as

$$F(\gamma_s) = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_s}{1+\gamma_s}}\right). \tag{10}$$

Now, we derive the optimal receiver. The likelihood for the user-1 is expressed as

$$p_{R_1|M_1}(r_1 \mid m_1) = \frac{1}{4}\frac{1}{\sqrt{2\pi N_0 / 2}}\sum_{m_2=1}^{4} e^{-\frac{\left(r_1 - |h_1| x_c(m_1, m_2)\right)^2}{2N_0/2}}. \tag{11}$$

The optimum detection is made, based on the maximum likelihood (ML), as

$$\hat{m}_1 = \underset{m_1 \in \{4,3,2,1\}}{\arg\max}\, p_{R_1|M_1}(r_1 \mid m_1). \tag{12}$$

The three exact decision boundaries are

$$r_1 = 0, \pm\left(2|h_1|\sqrt{\frac{\alpha P_A}{5}} + 3|h_1|\sqrt{\frac{2(1-\alpha)P_A}{25}}\right). \tag{13}$$

Then, the decision regions are given by

$$\begin{cases} m_1 = 4: & +\left(2|h_1|\sqrt{\frac{\alpha P_A}{5}} + 3|h_1|\sqrt{\frac{2(1-\alpha)P_A}{25}}\right) < r_1 \\ m_1 = 3: & 0 < r_1 < +\left(2|h_1|\sqrt{\frac{\alpha P_A}{5}} + 3|h_1|\sqrt{\frac{2(1-\alpha)P_A}{25}}\right) \\ m_1 = 2: & -\left(2|h_1|\sqrt{\frac{\alpha P_A}{5}} + 3|h_1|\sqrt{\frac{2(1-\alpha)P_A}{25}}\right) < r_1 < 0 \\ m_1 = 1: & r_1 < -\left(2|h_1|\sqrt{\frac{\alpha P_A}{5}} + 3|h_1|\sqrt{\frac{2(1-\alpha)P_A}{25}}\right). \end{cases} \tag{14}$$

Therefore, the SER performances for the user-1 is calculated by

$$P_e^{(1;\, M=4;\, ORDER\ NOMA;\, ML;\, optimal)} = 6 \cdot \frac{1}{4} \cdot \frac{1}{4} \sum_{j=0}^{3} F\left(\frac{P_A \left(j\sqrt{2(1-\alpha)/5} + \sqrt{\alpha} \right)^2 \Sigma_1 / 5}{N_0} \right) \tag{15}$$

Similarly, the likelihood for the user-2 is expressed as

$$p_{R_2|M_2}(r_2 \mid m_2) = \frac{1}{4} \frac{1}{\sqrt{2\pi N_0 / 2}} \sum_{m_1=1}^{4} e^{-\frac{\left(r_2 - |h_2| x_c(m_1, m_2)\right)^2}{2N_0/2}}. \tag{16}$$

The twelve approximate decision boundaries are

$$\begin{aligned} r_2 \simeq\ & +3|h_2|\sqrt{\frac{\alpha P_A}{5}} + \frac{9}{2}|h_2|\sqrt{\frac{2(1-\alpha)P_A}{25}}, +3|h_2|\sqrt{\frac{\alpha P_A}{5}} + \frac{11}{2}|h_2|\sqrt{\frac{2(1-\alpha)P_A}{25}}, \\ & +|h_2|\sqrt{\frac{\alpha P_A}{5}} + \frac{5}{2}|h_2|\sqrt{\frac{2(1-\alpha)P_A}{25}}, +3|h_2|\sqrt{\frac{\alpha P_A}{5}} + \frac{7}{2}|h_2|\sqrt{\frac{2(1-\alpha)P_A}{25}}, \\ & +|h_2|\sqrt{\frac{\alpha P_A}{5}} + \frac{1}{2}|h_2|\sqrt{\frac{2(1-\alpha)P_A}{25}}, +|h_2|\sqrt{\frac{\alpha P_A}{5}} + \frac{3}{2}|h_2|\sqrt{\frac{2(1-\alpha)P_A}{25}} \\ & -|h_2|\sqrt{\frac{\alpha P_A}{5}} - \frac{1}{2}|h_2|\sqrt{\frac{2(1-\alpha)P_A}{25}}, -|h_2|\sqrt{\frac{\alpha P_A}{5}} - \frac{3}{2}|h_2|\sqrt{\frac{2(1-\alpha)P_A}{25}}, \\ & -|h_2|\sqrt{\frac{\alpha P_A}{5}} - \frac{5}{2}|h_2|\sqrt{\frac{2(1-\alpha)P_A}{25}}, -3|h_2|\sqrt{\frac{\alpha P_A}{5}} - \frac{7}{2}|h_2|\sqrt{\frac{2(1-\alpha)P_A}{25}}, \\ & -3|h_2|\sqrt{\frac{\alpha P_A}{5}} - \frac{9}{2}|h_2|\sqrt{\frac{2(1-\alpha)P_A}{25}}, -3|h_2|\sqrt{\frac{\alpha P_A}{5}} - \frac{11}{2}|h_2|\sqrt{\frac{2(1-\alpha)P_A}{25}}. \end{aligned} \tag{17}$$

Therefore, the SER performance for the user-2 is calculated by

$$P_e^{(2;\, M=4;\, ORDER\ NOMA;\, ML;\, optimal)} \simeq 24 \cdot \frac{1}{4} \cdot \frac{1}{4} F\left(\frac{2(1-\alpha)P_A \Sigma_2 / 25}{N_0} \right). \tag{18}$$

V. Results and Discussions

Assume $\Sigma_1 = (2.0)^2$ and $\Sigma_2 = (0.9)^2$, and the constant total transmitted signal power to noise power ratio $P / N_0 = 40\ \mathrm{dB}$.

In the standard NOMA, the SIC is performed only on the user-1. Then the received signal is given by, if the perfect SIC is assumed

$$y_1 = r_1 - |h_1|\sqrt{(1-\alpha)P}s_2 = |h_1|\sqrt{\alpha P}s_1 + n_1. \tag{19}$$

Then the ideal perfect SIC SER performance of the user-1 is simply the SER performance of the 4PAM modulation

$$P_e^{(1;\, M=4;\, standard\ NOMA;\, perfect\ SIC;\, ideal)} = 6 \cdot \frac{1}{4} F\left(\frac{\alpha P_A \Sigma_1 / 5}{N_0} \right) \tag{20}$$

and the optimal ML SER performance $P_e^{(2;\, M=4;\, standard\ NOMA;\, ML;\, optimal)}$ of the user-2 is calculated with the conditional SER performance in [7].

As shown in Fig. 1, the ORDER NOMA achieves the perfect SIC SER performance of the standard NOMA, for the user-1, under Rayleigh fading channels.

As shown in Fig. 2, for the user-2, the ORDER NOMA fixes the seven performance degradations of

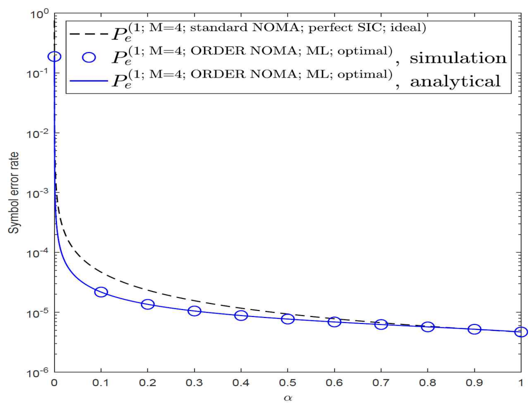

Fig. 1. Comparison of SERs for standard NOMA and ORDER NOMA for user-1.

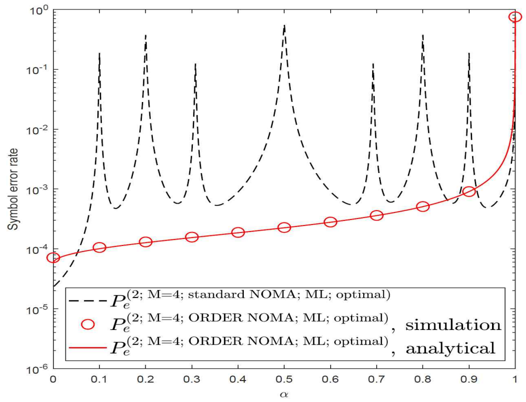

Fig. 2. Comparison of SERs for standard NOMA and ORDER NOMA for user-2.

the standard NOMA, under Rayleigh fading channels. Note that in the standard NOMA, for example, at $\alpha = 0.1$, the $m_1 = 4$ dominant term of $p_{R_2|M_2}(r_2 \mid m_2 = 3)$ is equal to the $m_1 = 1$ dominant term of $p_{R_2|M_2}(r_2 \mid m_2 = 4)$

$$x_s(m_1 = 4, m_2 = 3) = x_s(m_1 = 1, m_2 = 4) \\ = +|h_2|\sqrt{\frac{9}{10}P}\frac{2}{\sqrt{5}} \tag{21}$$

Therefore, the performance of the user-2 degrades severely, at $\alpha = 0.1$.

We also show the simulation results in Fig. 1 and 2, which are in good agreement with the analytical results. We observe a small approximation error in Fig. 2, for about $\alpha < 0.1$.

An additional observation on Fig. 2 is that the ORDER NOMA performs worse than the standard NOMA, for about $\alpha < 0.05$or $\alpha > 0.9$. One possible solution is that for such α, the standard SC can be used instead of the correlated SC.

Ⅵ. Conclusion

For 4-ary transmission, we presented the ORDER NOMA, with the correlated SC. It was shown that the ORDER NOMA not only achieves the perfect SIC SER performance, but also repairs the seven SER performance degradations in the standard 4PAM NOMA. As a consequence, the ORDER NOMA could be a promising SC for the 4-ary transmission NOMA. For the future researches, it is meaningful to design the correlated SC for more than two users.

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