## Correlated Superposition Coding for 3-User NOMA

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#### Abstract

In this letter, the authors propose the correlated superposition coding (SC) for 3-user non-orthogonal multiple access (NOMA). It is shown that the proposed SC not only achieves the near-perfect successive interference cancellation (SIC) bit error-rate (BER) performance, but also mitigates the performance degradation in the standard 3-user NOMA. In result, the correlated SC could be a promising SC for NOMA.

Key Words : NOMA, superposition coding, SIC, power allocation, correlation coefficient.


## I . Introduction

As a promising new radio (NR) mobile access technology, non-orthogonal multiple access (NOMA) has gained tremendous attention in the fifth generation (5G) mobile communication [1-6]. In NOMA, successive interference cancellation (SIC) is performed at the stronger channel users, to remove the inter-user interference (IUI) due to superposition coding (SC). We present the correlated SC, for the conventional 3-user binary pulse amplitude modulation (2PAM) NOMA.

- Our contributions: 1. The proposed scheme achieves the near-perfect SIC bit error-rate (BER) performance for the stronger channel users. 2. We mitigates the performance degradation for the weaker channel users in the standard 2PAM NOMA.


## II. System and Channel Model

Assume that the constant total transmitted power is $P$, and the power of the user- 1 signal $S_{1}$, the user-2 signal ${ }^{S_{2}}$, and the user-3 signal $s_{3}$ is normalized as $\mathbb{E}\left[\left|s_{1}\right|^{2}\right]=\mathbb{E}\left[\left|s_{2}\right|^{2}\right]=\mathbb{E}\left[\left|s_{3}\right|^{2}\right]=1$, with the correlation coefficients $\rho_{12}=\mathbb{E}\left[s_{1} s_{2}^{*}\right]$, $\rho_{13}=\mathbb{E}\left[s_{1} s_{3}^{*}\right]$, and $\rho_{23}=\mathbb{E}\left[s_{2} s_{3}^{*}\right]$. The power allocation factors are $\alpha$ and $\beta$ with $0 \leq \alpha, \beta, \alpha+\beta \leq 1$. Then the superimposed signal is expressed by

$$
\begin{equation*}
x=\sqrt{\alpha P_{A}} s_{1}+\sqrt{\beta P_{A}} s_{2}+\sqrt{(1-\alpha-\beta) P_{A}} s_{3} \tag{1}
\end{equation*}
$$

where the total allocated power $P_{A}$ is given by

$$
\begin{align*}
\frac{P}{P_{A}} & =1+2 \operatorname{Re}\left\{\rho_{12}\right\} \sqrt{\alpha} \sqrt{\beta}+2 \operatorname{Re}\left\{\rho_{13}\right\} \sqrt{\alpha} \sqrt{1-\alpha-\beta}  \tag{2}\\
& +2 \operatorname{Re}\left\{\rho_{23}\right\} \sqrt{\beta} \sqrt{1-\alpha-\beta} .
\end{align*}
$$

Before the SIC is performed on the user- 1 and user-2 with the better channel conditions, the received signals of the user- 1 , user- 2 , and user- 3 are represented as, for $i=1,2,3$

$$
\begin{align*}
& r_{i}= \\
& \left|h_{i}\right| \sqrt{\alpha P_{A} s_{1}}+\left|h_{i}\right| \sqrt{\beta P_{A} s_{2}}+\left|h_{i}\right| \sqrt{(1-\alpha-\beta) P_{A} s_{3}}+n_{i} \tag{3}
\end{align*}
$$

where $h_{1}, h_{2}$, and $h_{3}$ are the channel gains, with $\left|h_{1}\right|>\left|h_{2}\right|>\left|h_{3}\right|, \quad n_{1}, \quad n_{2}, \quad$ and $\quad n_{3} \sim \mathcal{N}\left(0, N_{0} / 2\right)$ are additive white Gaussian noise (AWGN), and $N_{6}$ is one-sided power spectral density. The notation $\mathcal{N}(\mu, \Sigma)$ denotes the normal distribution with mean $\mu$ and variance $\Sigma$.

## III. Correlated Superposition Coding

First, we start the independent SC in the standard 2PAM NOMA, and then design the correlated SC. Let the bits for the user- 1 , user- 2 , and user- 3 be

[^0]$m_{1}, m_{2}, m_{3} \in\{0,1\}$, with
\[

$$
\begin{equation*}
s_{1}, s_{2}, s_{3} \in\{+1,-1\} . \tag{4}
\end{equation*}
$$

\]

Then the independently superimposed signal in the standard 2PAM NOMA can be represented as

$$
\begin{align*}
& x_{s}\left(m_{1}, m_{2}, m_{2}\right)= \\
& (-1)^{m_{1}} \sqrt{\alpha P}+(-1)^{m_{2}} \sqrt{\beta P}+(-1)^{m_{3}} \sqrt{(1-\alpha-\beta) P} \tag{5}
\end{align*}
$$

Note that the order of the 8 superimposed signals changes, dependent on the power allocation.

Now, we design the user- 2 and user- 3 signal, in the power domain, dependent on the standard 2PAM user-1 signal. Thus, the correlated SC is represented by

$$
\begin{align*}
& x_{c}\left(m_{1}, m_{2}, m_{3}\right)= \\
& (-1)^{m_{1}} \sqrt{\alpha P_{A}}+(-1)^{m_{1}} m_{2} \sqrt{2 \beta P_{A}}  \tag{6}\\
& +(-1)^{m_{1}}\left(m_{2}+\left(m_{2} \oplus m_{3}\right)\right) \sqrt{2(1-\alpha-\beta) P_{A} / 3} .
\end{align*}
$$

The motivation of the correlated SC is as follows in detail. For the standard independent SC NOMA, for example, the amplitude of the independently superimposed signal is reduced

$$
\begin{align*}
& x_{s}\left(m_{1}=0, m_{2}=1, m_{3}=1\right) \\
& =+\sqrt{\alpha P_{A}}-\sqrt{\beta P_{A}}-\sqrt{(1-\alpha-\beta) P_{A}} . \tag{7}
\end{align*}
$$

Therefore, the correlated SC is designed for the amplitude not to be reduced

$$
\begin{align*}
& x_{c}\left(m_{1}=0, m_{2}=1, m_{3}=1\right) \\
& =+\sqrt{\alpha P_{A}}+\sqrt{2 \beta P_{A}}+\sqrt{2(1-\alpha-\beta) P_{A} / 3} . \tag{8}
\end{align*}
$$

Thus, the SIC BER performance of the user-1 is not degraded by SC. In addition, in order to mitigate the severe performance degradation for the user-3, the bit-to-symbol mapping is carefully designed, for example,

$$
\begin{align*}
& x_{c}\left(m_{1}=0, m_{2}=1, m_{3}=0\right)=+\sqrt{\alpha P_{A}}+\sqrt{2 \beta P_{A}}+2 \sqrt{2(1-\alpha-\beta) P_{A} / 3} \\
& x_{c}\left(m_{1}=0, m_{2}=1, m_{3}=1\right)=+\sqrt{\alpha P_{A}}+\sqrt{2 \beta P_{A}}+\sqrt{2(1-\alpha-\beta) P_{A} / 3} \\
& x_{c}\left(m_{1}=0, m_{2}=0, m_{3}=1\right)=+\sqrt{\alpha P_{A}}+0+\sqrt{2(1-\alpha-\beta) P_{A} / 3} \\
& x_{c}\left(m_{1}=0, m_{2}=0, m_{3}=0\right)=+\sqrt{\alpha P_{A}}+0+0 . \tag{9}
\end{align*}
$$

Note that the $x_{c}\left(m_{1}=0, m_{2}=0, m_{3}=1\right)$ and $x_{c}\left(m_{1}=0, m_{2}=1, m_{3}=1\right)$ for the user-3 bit $m_{3}=1$ do not interfere each other, which prevents the severe performance degradation in the standard NOMA. And for the user-2, above two design rules are applied simultaneously.

## IV. BER Derivation

The correlation coefficients of the correlated SC are calculated by

$$
\begin{align*}
\rho_{12} & =\mathbb{E}\left[s_{1} s_{2}^{*}\right] \\
& =\frac{1}{8}(1 \times \sqrt{2}+1 \times \sqrt{2}+0+0) \\
& +\frac{1}{8}(0+0+(-1) \times(-\sqrt{2})+(-1) \times(-\sqrt{2})) \\
& =\frac{1}{8}(4 \sqrt{2})  \tag{10}\\
& =\frac{\sqrt{2}}{2} \\
& =\frac{1}{\sqrt{2}} \simeq 0.7071, \\
\rho_{13} & =\mathbb{E}\left[s_{1} s_{3}^{*}\right] \\
& =\frac{1}{8}(0+1 \times \sqrt{2 / 3}+1 \times \sqrt{2 / 3}+1 \times 2 \sqrt{2 / 3}) \\
& +\frac{1}{8}(0+(-1) \times(-\sqrt{2 / 3})) \\
& +\frac{1}{8}((-1) \times(-\sqrt{2 / 3})+(-1) \times(-2 \sqrt{2 / 3}))  \tag{11}\\
& =\frac{1}{8}(8 \sqrt{2 / 3}) \\
& =\sqrt{2 / 3} \simeq 0.8165,
\end{align*}
$$

and

$$
\begin{align*}
\rho_{23} & =\mathbb{E}\left[s_{2} s_{3}^{*}\right] \\
& =\frac{1}{8}(0+0+\sqrt{2} \times \sqrt{2 / 3}+\sqrt{2} \times 2 \sqrt{2 / 3}) \\
& +\frac{1}{8}(0+0+(-\sqrt{2}) \times(-\sqrt{2 / 3})+(-\sqrt{2}) \times(-2 \sqrt{2 / 3}))  \tag{12}\\
& =\frac{1}{8}(6 \times 2 / \sqrt{3}) \\
& =\frac{1}{2} \sqrt{3} \simeq 0.8660 .
\end{align*}
$$

Consider Rayleigh fading channels with $\mathbb{E}\left[\left|h_{1}\right|^{2}\right]=\Sigma_{1}, \mathbb{E}\left[\left|h_{2}\right|^{2}\right]=\Sigma_{2}$, and $\mathbb{E}\left[\left|h_{3}\right|^{2}\right]=\Sigma_{3}$. For the
simplification of Rayleigh fading BER performance, we define the notation as

$$
\begin{equation*}
F\left(\gamma_{b}\right)=\frac{1}{2}\left(1-\sqrt{\frac{\gamma_{b}}{1+\gamma_{b}}}\right) . \tag{13}
\end{equation*}
$$

Now, we derive the optimal receiver. The likelihood for the user-1 is expressed as

$$
\begin{equation*}
p_{R_{1} \mid M_{1}}\left(r_{1} \mid m_{1}\right)=\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} \sum_{m_{2}=0}^{1} \sum_{m_{3}=0}^{1} e^{-\frac{\left(r_{1}-\left|m_{1}\right| x_{2}\left(m_{1}, m_{2}, m_{3}\right)\right)^{2}}{2 N_{0} / 2}} . \tag{14}
\end{equation*}
$$

The optimum detection is made, based on the maximum likelihood (ML), as

$$
\begin{equation*}
\hat{m}_{1}=\underset{m_{1} \in\{0,1\}}{\arg \max } p_{R_{1} \mid M_{1}}\left(r_{1} \mid m_{1}\right) \tag{15}
\end{equation*}
$$

The one exact decision boundary is

$$
\begin{equation*}
r_{1}=0 \tag{16}
\end{equation*}
$$

Then, the decision regions are given by

$$
\begin{cases}m_{1}=0: \quad 0<r_{1}  \tag{17}\\ m_{1}=1: & r_{1}<0 .\end{cases}
$$

Therefore, the BER performance for the user-1 is calculated by

$$
\begin{align*}
& P_{e}^{(1 ; \text { correlated NOMA; } M L ; \text { optimal })}=\frac{1}{4} \sum_{m_{2}=0}^{1} \sum_{m_{3}=0}^{1}  \tag{18}\\
& F\left(\frac{P_{A}\left(\left(m_{2}+\left(m_{2} \oplus m_{3}\right)\right) \sqrt{2(1-\alpha-\beta) / 3}+m_{2} \sqrt{2 \beta}+\sqrt{\alpha}\right)^{2} \Sigma_{1}}{N_{0}}\right)
\end{align*}
$$

Similarly, the likelihood for the user-2 is expressed as

$$
\begin{equation*}
p_{R_{2} \mid M_{2}}\left(r_{2} \mid m_{2}\right)=\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} \sum_{m_{1}=0}^{1} \sum_{m_{3}=0}^{1} e^{-\frac{\left(r_{1}-\left|m_{1}\right| x_{c}\left(m_{1}, m_{2}, m_{3}\right)\right)^{2}}{2 N_{0} / 2}} . \tag{19}
\end{equation*}
$$

The two approximate decision boundaries are

$$
\begin{equation*}
r_{2}= \pm\left|h_{2}\right| \sqrt{\alpha P_{A}} \pm \frac{\left|h_{2}\right| \sqrt{2 \beta P_{A}}}{2} \pm\left|h_{2}\right| \sqrt{2(1-\alpha-\beta) P_{A} / 3} . \tag{20}
\end{equation*}
$$

Then, the decision regions are given by

$$
\left\{\begin{align*}
m_{2}=0: & r_{2}<+\left|h_{2}\right| \sqrt{\alpha P_{A}}+\frac{\left|h_{2}\right| \sqrt{2 \beta P_{A}}}{2}+\left|h_{2}\right| \sqrt{2(1-\alpha-\beta) P_{A} / 3},  \tag{21}\\
& -\left|h_{2}\right| \sqrt{\alpha P_{A}}-\frac{\left|k_{2}\right| \sqrt{2 \beta P_{A}}}{2}-\left|h_{2_{2}}\right| \sqrt{2(1-\alpha-\beta) P_{A} / 3}<r_{2} . \\
m_{2}=1: & r_{2}>+\left|h_{2}\right| \sqrt{\alpha P_{A}}+\frac{\left|h_{2}\right| \sqrt{2 \beta P_{A}}}{2}+\left|h_{2}\right| \sqrt{2(1-\alpha-\beta) P_{A} / 3}, \\
& r_{2}<-\left|h_{2}\right| \sqrt{\alpha P_{A}}-\frac{\left|h_{2}\right| \sqrt{2 \beta P_{A}}}{2}-\left|h_{2}\right| \sqrt{2(1-\alpha-\beta) P_{A} / 3} .
\end{align*}\right.
$$

Therefore, the BER performance for the user-2 is calculated by

$$
\begin{align*}
& P_{e}^{(2 ; \text { correlated NOMA; ML; optimal) })} \\
& \simeq \frac{1}{2} F\left(\frac{P_{A}(\sqrt{\beta / 2})^{2} \Sigma_{2}}{N_{0}}\right) . \tag{22}
\end{align*}
$$

Remark that there are two causes of the approximation; the initial cause is the decision boundary approximation, and the secondary cause is the dominant term approximation. The decision boundary approximation is tolerated by the following observation; the approximate decision boundary $r_{2}=+\left|h_{2}\right| \sqrt{\alpha P_{A}}+\frac{\left|h_{2}\right| \sqrt{2 \beta P_{A}}}{2}+\left|h_{2}\right| \sqrt{2(1-\alpha-\beta) P_{A} / 3}$ is obtained from the equation

$$
\begin{equation*}
p_{R_{2} \mid M_{2}}\left(r_{2} \mid m_{2}=0\right)=p_{R_{2} \mid M_{2}}\left(r_{2} \mid m_{2}=1\right) \tag{23}
\end{equation*}
$$

which is given by

$$
\begin{align*}
& \frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left(r_{1}-\left|h_{1}\right|\left(+\sqrt{\alpha P_{A}}+0+\sqrt{2(1-\alpha-\beta) P_{A} / 3}\right)\right)^{2}}{2 N_{0} / 2}} \\
& +\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left(r_{1}-\left|h_{1}\right|\left(+\sqrt{\alpha P_{A}}+0+0\right)\right)^{2}}{2 N_{0} / 2}} \\
& +\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left(r_{1}-\left|h_{1}\right|\left(-\sqrt{\alpha P_{A}}-0-0\right)\right)^{2}}{2 N_{0} / 2}} \\
& +\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left(r_{1}-\left|h_{1}\right|\left(-\sqrt{\alpha P_{A}}-0-\sqrt{2(1-\alpha-\beta) P_{A} / 3}\right)\right)^{2}}{2 N_{0} / 2}} \\
& =\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left(r_{1}-\left|h_{1}\right|\left(+\sqrt{\alpha P_{A}}+\sqrt{2 \beta P_{A}}+2 \sqrt{2(1-\alpha-\beta) P_{A} / 3}\right)\right)^{2}}{2 N_{0} / 2}}  \tag{24}\\
& +\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left(r_{1}-\left|h_{1}\right|\left(+\sqrt{\alpha P_{A}}+\sqrt{2 \beta P_{A}}+\sqrt{2(1-\alpha-\beta) P_{A} / 3}\right)\right)^{2}}{2 N_{0} / 2}} \\
& +\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left(r_{1}-\left|h_{1}\right|\left(-\sqrt{\alpha P_{A}}-\sqrt{2 \beta P_{A}}-\sqrt{2(1-\alpha-\beta) P_{A} / 3}\right)\right)^{2}}{2 N_{0} / 2}} \\
& +\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left(r_{1}-\left|h_{1}\right|\left(-\sqrt{\alpha P_{A}}-\sqrt{2 \beta P_{A}}-2 \sqrt{2(1-\alpha-\beta) P_{A} / 3}\right)\right)^{2}}{2 N_{0} / 2}}
\end{align*}
$$

Note that, at

$$
\begin{align*}
r_{2} & =+\left|h_{2}\right| \sqrt{\alpha P_{A}}+\frac{\left|h_{2}\right| \sqrt{2 \beta P_{A}}}{2}+\left|h_{2}\right| \sqrt{2(1-\alpha-\beta) P_{A} / 3} \\
& \frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left(r_{1}-\left|h_{1}\right|\left(+\sqrt{\alpha P_{A}}+0+\sqrt{2(1-\alpha-\beta) P_{A} / 3}\right)\right)^{2}}{2 N_{0} / 2}} \\
& +\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left.\left(r_{1}-\left|h_{1}\right| \mid+\sqrt{\alpha P_{A}}+0+0\right)\right)^{2}}{2 N_{0} / 2}} \\
& =  \tag{25}\\
& \frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left.\left(r_{1}-\left|h_{1}\right| \mid+\sqrt{\alpha P_{A}}+\sqrt{2 \beta P_{A}}+2 \sqrt{2(1-\alpha-\beta) P_{A} / 3}\right)\right)^{2}}{2 N_{0} / 2}} . \\
& +\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left.\left(r_{1}-\left|h_{1}\right| \mid+\sqrt{\alpha P_{A}}+\sqrt{2 \beta P_{A}}+\sqrt{2(1-\alpha-\beta) P_{A} / 3}\right)\right)^{2}}{2 N_{0} / 2}} .
\end{align*}
$$

And the four terms approximated to zero are so small, that we could not obtain the numerical values, for $P / N_{0}=40 \mathrm{~dB}$ in our case. Instead, for $P / N_{0}=20 \mathrm{~dB}=100$, we give the numerical values, at $\quad r_{2}=+\left|h_{2}\right| \sqrt{\alpha P_{A}}+\frac{\left|h_{2}\right| \sqrt{2 \beta P_{A}}}{2}+\left|h_{2}\right| \sqrt{2(1-\alpha-\beta) P_{A} / 3}$, with $\alpha=0.1, \beta=0.2$, and $\left|h_{2}\right|=1.0$, as follows
$+\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left(r_{1}-\left|h_{1}\right|\left(-\sqrt{a P_{A}}-0-0\right)\right)^{2}}{2 N_{0} / 2}} \simeq 1.9049 \times 10^{-52}$,
$+\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left(r_{1}-\left|h_{1}\right|\left(-\sqrt{\alpha P_{A}}-0-\sqrt{2(1-\alpha-\beta) P_{A} / 3}\right)\right)^{2}}{2 N_{0} / 2}} \simeq 8.4329 \times 10^{-104}$,
$+\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left(r_{1}-\left|h_{1}\right| \mid\left(-\sqrt{\alpha P_{A}}-\sqrt{2 \beta P_{A}}-\sqrt{2(1-\alpha-\beta) P_{A} / 3}\right)\right)^{2}}{2 N_{0} / 2}} \simeq 3.4147 \times 10^{-167}$,
$+\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} e^{-\frac{\left(r_{1}-\left|h_{1}\right|\left(-\sqrt{\alpha P_{A}}-\sqrt{2 \beta P_{A}}-2 \sqrt{2(1-\alpha-\beta) P_{A} / 3}\right)\right)^{2}}{2 N_{0} / 2}} \simeq 8.7810 \times 10^{-253}$.

Therefore, the decision boundary approximation is tolerable. Next, we consider the dominant term approximation. Based on the approximated decision boundaries, the exact $P_{e}^{(2 ; \text { correlated NOMA; } M L ; \text { optimal })}$ is given by

$$
\begin{align*}
& P_{e}^{(2 ; \text { correlated NOMA } ; \text { ML } ; \text { optimal })} \\
& =\frac{1}{4} f\left(0 ; \frac{1}{2} ; 1\right)-\frac{1}{4} f\left(2 ; \frac{3}{2} ; 3\right) \\
& +\frac{1}{4} f\left(0 ; \frac{1}{2} ; 0\right)-\frac{1}{4} f\left(2 ; \frac{3}{2} ; 2\right)  \tag{27}\\
& +\frac{1}{4} f\left(0 ; \frac{1}{2} ; 0\right)+\frac{1}{4} f\left(2 ; \frac{1}{2} ; 2\right) \\
& +\frac{1}{4} f\left(0 ; \frac{1}{2} ; 1\right)+\frac{1}{4} f\left(2 ; \frac{1}{2} ; 1\right)
\end{align*}
$$

where

$$
\begin{align*}
& f(A ; B ; C) \\
& =F\left(\frac{P_{A}\left(A \sqrt{\alpha P_{A}}+B \sqrt{2 \beta P_{A}}+C \sqrt{2(1-\alpha-\beta) P_{A} / 3}\right)^{2} \Sigma_{2}}{N_{0}}\right) . \tag{28}
\end{align*}
$$

For $P / N_{0}=40 \mathrm{~dB}$ in our case, with $\alpha=0.1$, $\beta=0.2$, and $\Sigma_{2}=(1.0)^{2}$, we give the numerical values, as follows

$$
\begin{align*}
& \frac{1}{4} f\left(0 ; \frac{1}{2} ; 0\right) \simeq 1.4226 \times 10^{-4} \\
& \frac{1}{4} f\left(0 ; \frac{1}{2} ; 1\right) \simeq 1.4267 \times 10^{-5} \\
& \frac{1}{4} f\left(2 ; \frac{1}{2} ; 1\right) \simeq 5.3514 \times 10^{-6} \\
& \frac{1}{4} f\left(2 ; \frac{1}{2} ; 2\right) \simeq 2.6591 \times 10^{-6}  \tag{29}\\
& \frac{1}{4} f\left(2 ; \frac{3}{2} ; 2\right) \simeq 1.6404 \times 10^{-6} \\
& \frac{1}{4} f\left(2 ; \frac{3}{2} ; 3\right) \simeq 1.0812 \times 10^{-6} .
\end{align*}
$$

Based on the above observations, we approximate the BER as the two dominant terms, in the equation (22).

Similarly, the likelihood for the user-3 is expressed as

$$
\begin{equation*}
p_{R_{3} \mid M_{3}}\left(r_{3} \mid m_{3}\right)=\frac{1}{4} \frac{1}{\sqrt{2 \pi N_{0} / 2}} \sum_{m_{1}=0}^{1} \sum_{m_{2}=0}^{1} e^{-\frac{\left(r_{1}\left|m_{1}\right| x_{c}\left(m_{1}, m_{2}, m_{3}\right)\right)^{2}}{2 N_{0} / 2}} . \tag{30}
\end{equation*}
$$

The four approximate decision boundaries are

$$
\begin{align*}
r_{3}= & \pm\left|h_{3}\right| \sqrt{\alpha P_{A}} \pm 0 \pm \frac{1}{2}\left|h_{3}\right| \sqrt{2(1-\alpha-\beta) P_{A} / 3}  \tag{31}\\
& \pm\left|h_{3}\right| \sqrt{\alpha P_{A}} \pm\left|h_{3}\right| \sqrt{2 \beta P_{A}} \pm \frac{3}{2}\left|h_{3}\right| \sqrt{2(1-\alpha-\beta) P_{A} / 3}
\end{align*}
$$

Therefore, the BER performance for the user-3 is calculated by

$$
\begin{align*}
& P_{e}^{(3 ; \text { correlated NOMA; ML; optimal })} \\
& \simeq F\left(\frac{P_{A}(\sqrt{(1-\alpha-\beta) / 3 / 2})^{2} \Sigma_{3}}{N_{0}}\right) . \tag{32}
\end{align*}
$$

## V. Results and Discussions

Assume $\quad \Sigma_{1}=(1.1)^{2}, \quad \Sigma_{2}=(1.0)^{2}$ and $\Sigma_{3}=(0.9)^{2}$, and the constant total transmitted signal power to noise power ratio $P / N_{0}=40 \mathrm{~dB}$, where $\Sigma_{1}, \Sigma_{2}$, and $\Sigma_{3}$ are chosen for the average standard deviation to be one, and $P / N_{0}$ is selected for the BER performance to be $10^{-4}<P_{o}<10^{-3}$.

In the standard NOMA, SIC is performed on the user-1 and user-2. Then the received signal is given by, if the perfect SIC is assumed,

$$
\begin{align*}
& y_{1}=\left|h_{1}\right| \sqrt{\alpha P} s_{1}+n_{1} \\
& y_{2}=\left|h_{2}\right| \sqrt{\beta P} s_{2}+\left(\left|h_{2}\right| \sqrt{\alpha P} s_{1}+n_{2}\right) . \tag{33}
\end{align*}
$$

Then the ideal perfect SIC BER performance of the user-1 is simply the BER performance of the 2PAM modulation

$$
\begin{align*}
& P_{e}^{(1 ; \text { standard NOMA; perfect SIC } ; \text { ideal })} \\
& =F\left(\frac{\alpha P \Sigma_{1}}{N_{0}}\right) \tag{34}
\end{align*}
$$

The ideal perfect SIC plus maximum likelihood (ML) BER performance $P_{e}^{(2 ; \text { standard NOMA; perfect SIC ideal }+ \text { ML optimal })}$ of the user-2 is given in [7] and the optimal ML BER performance $P_{e}^{(3 ; \text { standard NOMA; ML; optimal })}$ of the user-3 is calculated with the conditional BER performance in [8].

As shown in Fig. 1, the correlated SC NOMA achieves the near-perfect SIC BER performance of the standard NOMA, for the user-1. For the user-2, as shown in Fig. 2, the correlated SC NOMA achieves the near-perfect SIC BER and mitigates the performance degradation of the standard NOMA. As shown in Fig. 3, for the user-3, the correlated SC NOMA mitigates the severe performance degradation of the standard NOMA.

As shown in Fig .2, the severe BER performance degradation of the user-2 occurs the following power


Fig. 1. Comparison of BERs for standard NOMA and correlated NOMA for user-1.


Fig. 2. Comparison of BERs for standard NOMA and correlated NOMA for user-2.


Fig. 3. Comparison of BERs for standard NOMA and correlated NOMA for user-3.
allocation coordinate $(\alpha, \beta)$,

$$
\begin{equation*}
\alpha=\beta, \quad 0 \leq \alpha+\beta \leq 1 \tag{35}
\end{equation*}
$$

Table 1. Quantitative BER Performance Mitigation

| $(\alpha, \beta)$ | $(0.1,0.1)$ | $(0.3,0.3)$ | $(0.5,0.5)$ |
| :--- | :---: | :---: | :---: |
| $P_{e}^{(2 ; \text { standard NOMA; perfect SIC ideal }+ \text { ML optimal })}$ | 0.25 | 0.25 | 0.25 |
|  | $\Downarrow$ | $\Downarrow$ | $\Downarrow$ |
| $P_{e}^{(2 ; \text { correlated NOMA; ML; optimal })}$ | $5.2166 \times 10^{-4}$ | $2.1555 \times 10^{-4}$ | $8.5311 \times 10^{-5}$ |

Therefore, in Fig. 2, the mitigation of the severe BER performance degradation of the user-2 is dependent on the power allocation coordinate $(\alpha, \beta)$, which is tabulated in Table 1 , quantitatively.

In addition, for the comparison under the various channel environments, we depict the BER


Fig. 4. Comparison of BERs for standard NOMA and correlated NOMA for user-1.


Fig. 5. Comparison of BERs for standard NOMA and correlated NOMA for user-2.
performances with $\quad \Sigma_{1}=(2.1)^{2}, \Sigma_{2}=(0.8)^{2}$ and $\Sigma_{3}=(0.1)^{2}$. As shown in Fig. 4, Fig. 5, and Fig. 6, we observe the similar performances.


Fig. 6. Comparison of BERs for standard NOMA and correlated NOMA for user-3.

## VI. Conclusion

We presented the correlated SC, for 3-user NOMA. It was shown that the correlated SC NOMA achieves not only the near-perfect SIC BER performance, but also mitigates the performance degradation in the standard NOMA. As a consequence, the correlated SC could be a promising SC for SIC and IUI suppression.

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