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On BER for Low Complexity Non-SIC NOMA with *M*-User

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ABSTRACT

In non-orthogonal multiple access (NOMA), one of factors for implementation complexity is successive interference cancellation (SIC), which is essential for each user to operate at quality of service (QoS). In this letter, we consider the NOMA scheme with M-user, without the SIC complexity. First, we derive the closed-form BER expression. Then it is shown that the non-SIC NOMA BER is comparable to the SIC NOMA BER. This can be possible, with two reasons; one is the power allocation of the user fairness, and the other is the optimal maximum-likelihood (ML) detection.

Key Words : NOMA, user fairness, superposition coding, SIC, power allocation

Ⅰ. Introduction

In the fifth generation (5G) mobile networks, non-orthogonal multiple access (NOMA) has been considered as a promising multiple access scheme [1-6]. Recently, the bit-error rate (BER) for *M*-user NOMA has been derived[7]. In this letter, we consider the NOMA scheme, without successive interference cancellation (SIC). We derive an analytical expression for BER, and show that the BER of this proposed scheme is compared to that of the conventional SIC scheme.

Ⅱ. System and Channel Model

We consider a cellular downlink NOMA transmission system, in which a base station and M users within the cell. The Rayleigh fading channel between the mth user and the base station is denoted by $h_m \sim \mathcal{CN}(0,\Sigma_m)$, $1 \le m \le M$. The base station will send the superimposed signal $x=\sum_{m=1}^{M}\sqrt{\alpha_m P}s_m$, where s_m is the message for the mth user, α_m is the power allocation coefficient, with $\sum_{m=1}^{M}\alpha_m=1$, $\alpha_1 \le \cdots \le \alpha_M$, and P is the constant total transmitted power at the base station. The power of s_m is normalized as unit power, $\mathbb{E}[s_m s_m^*]=\mathbb{E}[|s_m|^2]=1$. The observation at the mth user is given by

$$y_m = h_m x + w_m, \tag{1}$$

where $w_m \sim \mathcal{CN}(0,N_0)$ is additive white Gaussian noise (AWGN). We consider the binary phase shift keying (BPSK) modulation, with $s_m \in \{+1,-1\}$. Then the following metric is the sufficient statistics

$$r_m = |h_m| x + n_m, \tag{2}$$

where $n_m \sim \mathcal{N}(0,N_0/2)$. Let the information input bit for the mth user be $b_m \in \{0,1\}$. Then, the bit-to-symbol mapping is given as

$$\begin{cases} s_m(b_m=0)=+1 \\ s_m(b_m=1)\ =-1 \end{cases}, \quad \text{or} \quad s_m(b_m)=(-1)^{b_m}. \tag{3}$$

And we use the binary representation notation for the index i, $0 \le i \le 2^{m-1}-1$,

$$(i)_2 = b_{m-1}b_{m-2}\cdots b_2 b_1 \tag{4}$$

where each bit b_j, $1 \le j \le m-1$, corresponds to the jth user. In [7], the BER performance of *N*-user NOMA with the perfect successive interference cancellation (SIC) and maximum likelihood (ML)

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detection was derived, when the channels are sorted by the instantaneous realizations, i.e., $|h_1| \geq \cdots \geq |h_M|$. In this letter, under the sort by the channel gain variances, i.e., $\Sigma_1 \geq \cdots \geq \Sigma_M$, the BER of the mth user can be expressed by

$$P_e^{(m;\,\text{SIC NOMA})} = \sum_{i=0}^{2^{m-1}-1} \frac{1}{2^{m-1}} F\left(\frac{\Sigma_m P \alpha_{m,i}}{N_0}\right) \tag{5}$$

where $\alpha_{m,i}$ is given by

$$\alpha_{m,i} = \left(\sqrt{\alpha_m} + (-1)^{b_{m-1}}\sqrt{\alpha_{m-1}} + \ldots + (-1)^{b_1}\sqrt{\alpha_1}\right)^2 \tag{6}$$

and for the compact presentation of Rayleigh fading BER performance, we use the notation as

$$F(\gamma_b) = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_b}{1+\gamma_b}}\right). \tag{7}$$

Further, the compact presentation of BER performance, we use the notation as

$$P_e^{(m;\,\text{SIC NOMA})}(I;C) = \sum_{i=I}^{2^{m-1}-1} \frac{1}{2^{m-1}} F\left(\frac{\Sigma_C P \alpha_{m,i}}{N_0}\right) \tag{8}$$

where I is the lower limit of the summation index, and C is the channel index. With the above notation,

$$P_e^{(m;\,\text{SIC NOMA})} = P_e^{(m;\,\text{SIC NOMA})}(0;m). \tag{9}$$

Ⅲ. BER Derivation for Non-SIC NOMA with *M*-user

Now, we derive the BER expression for NOMA without SIC. The decoding strategy in this letter is based on the ML detection; specifically, the non-SIC NOMA receiver detects the information bits from the received signal r_m, i.e., before the SIC is performed at each receiver. First, for $m = M$, if $\alpha_{M,2^{M-1}-1} \geq 0$, the decision boundary is only the one, i.e., $r_M^{(\text{db};\,1st;\,\text{non-SIC NOMA})} = 0$. Otherwise, the number of the decision boundaries is more than one. Thus the condition for only $r_M^{(\text{db};\,1st;\,\text{non-SIC NOMA})} = 0$ is obtained by

$$\begin{aligned} &\alpha_{M,2^{M-1}-1} = \left(\sqrt{\alpha_M} - \sqrt{\alpha_{M-1}} - \ldots - \sqrt{\alpha_1}\right)^2 \geq 0 \\ &\therefore \sqrt{\alpha_M} \geq \textstyle\sum_{i=1}^{M-1} \sqrt{\alpha_i} \end{aligned} \tag{10}$$

Then, with the above assumption, the non-SIC NOMA BER of the Mth user equals to the standard SIC NOMA BER

$$P_e^{(M;\,\text{non-SIC NOMA})} = P_e^{(M;\,\text{SIC NOMA})} \tag{11}$$

because in the standard SIC NOMA, the weakest channel user does not perform SIC.

Note that for $1 \leq m \leq M-1$, the number of the decision boundaries is more than one. Thus, in order to avoid confusion, we define formally the nth nearest decision boundary for the mth user

$$r_m^{(\text{db};\,nth;\,\text{SIC NOMA } or \text{ non-SIC NOMA})} \tag{12}$$

Further, for the compact presentation, we introduce the normalized decision boundary

$$r_{m,\text{norm}}^{(\text{db};\,nth;\,\text{SIC NOMA } or \text{ non-SIC NOMA})} = \frac{r_m^{(\text{db};\,nth;\,\text{SIC NOMA } or \text{ non-SIC NOMA})}}{|h_m|\sqrt{P}} \tag{13}$$

We depict the nth nearest decision boundary, in Fig. 1, for the SIC NOMA and the non-SIC NOMA, with $M = 4$, respectively.

Note that near $\sqrt{\alpha_M} \simeq \sum_{i=1}^{M-1}\sqrt{\alpha_i}$, i.e.,

$$\begin{aligned} &+\sqrt{\alpha_M} - \sqrt{\alpha_{M-1}} - \ldots - \sqrt{\alpha_1} \\ &\simeq -\sqrt{\alpha_M} + \sqrt{\alpha_{M-1}} + \ldots + \sqrt{\alpha_1}, \end{aligned} \tag{14}$$

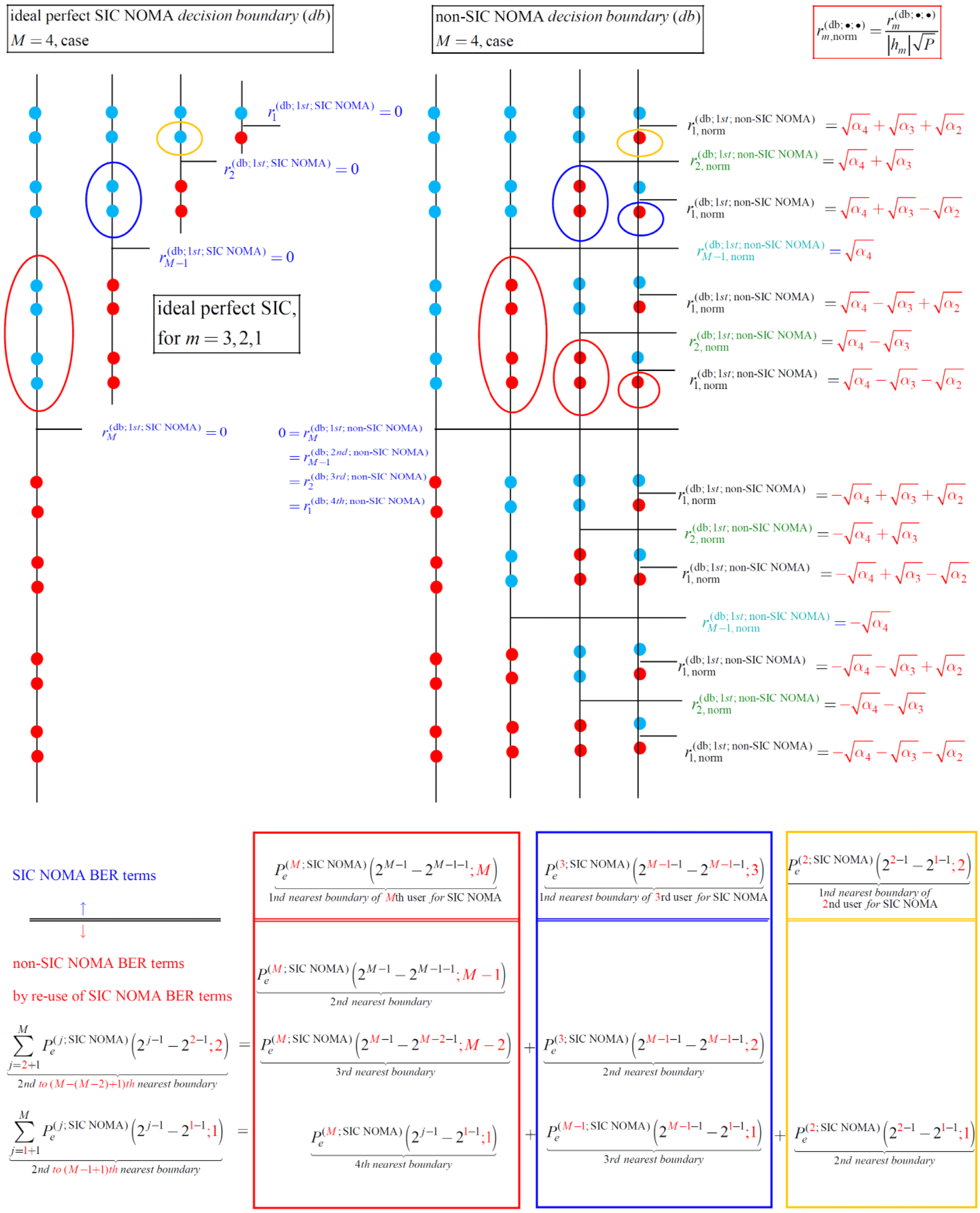

Fig. 1. Non-SIC NOMA BER derivation with $2nd$ to $(M-m+1)th$ nearest boundary.

$P_e^{(M;\text{SIC NOMA})}$ degrades severely, which requires $\sqrt{\alpha_M} \gg \sum_{i=1}^{M-1}\sqrt{\alpha_i}$. Such assumption makes $r_M^{(\text{db};1st;\text{non-SIC NOMA})}=0$ as the $2nd$ nearest decision boundary $r_{M-1}^{(\text{db};2nd;\text{non-SIC NOMA})}=0$, for $P_e^{(M-1;\text{non-SIC NOMA})}$. For $m=M-1$, the two $1st$ nearest decision boundaries, $r_{M-1}^{(\text{db};1st;\text{non-SIC NOMA})}=\pm|h_{M-1}|\sqrt{P}\sqrt{\alpha_M}$, of the non-SIC NOMA corresponds to the $1st$ nearest decision boundary, $r_M^{(\mathrm{db};1st;\text{SIC NOMA})}=0$, of the standard SIC NOMA. Then, the non-SIC NOMA BER of the $(M-1)$th user is given by

$$P_e^{(M-1;\,\text{non-SIC NOMA})} \simeq \underbrace{P_e^{(M-1;\,\text{SIC NOMA})}}_{1st\ nearest\ boundary} + \underbrace{P_e^{(M;\,\text{SIC NOMA})}\left(2^{M-1}-2^{M-1-1};M-1\right)}_{2nd\ nearest\ boundary}. \tag{15}$$

Similarly, for $1\le m\le M-1$, the non-SIC NOMA BER of the mth user is derived as

$$P_e^{(m;\,\text{non-SIC NOMA})} \simeq \underbrace{P_e^{(m;\,\text{SIC NOMA})}}_{1st\ nearest\ boundary} + \underbrace{\sum_{j=m+1}^{M} P_e^{(j;\,\text{SIC NOMA})}\left(2^{j-1}-2^{m-1};m\right)}_{2nd\ to\ (M-m+1)th\ nearest\ boundary}. \tag{16}$$

Note that the approximation up to the $(M-m+1)th$ nearest decision boundary is tolerable, because from the $(M-m+2)th$ nearest decision boundary, the contribution to BER is little. The validation of such approximation is presented in Appendix, in detail with the exact BER for $M=3$ and the 2nd user, which is compared to the approximated BER in this letter.

Ⅳ. Results and Discussions

We consider all the three users within the cell with $M=3$. Assume $\Sigma_1=1.5$, $\Sigma_2=1.0$, and $\Sigma_3=0.5$, and the constant total transmitted signal power to noise power ratio $P/N_0=40$ dB.

In Fig. 2, we plot the BER for the 3rd user, with $\alpha_3 \ge \frac{2}{3} \simeq 0.67$. In such range, the BER expression holds, regardless any combination of (α_1,α_2), under $\alpha_1+\alpha_2 \le \frac{1}{3}$, and to avoid the severe BER degradation, the more power is required, i.e.,

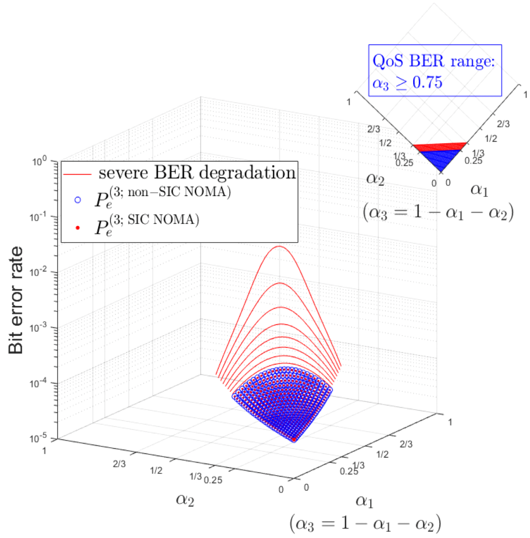

Fig. 2. BER, for QoS and degradation ranges for 3rd user.

$\alpha_3 \ge 0.75$. As shown in Fig. 1, the BER performance of the non-SIC NOMA is comparable to that of the standard SIC NOMA, because $P_e^{(m;\,\text{non-SIC NOMA})}=P_e^{(m;\,\text{SIC NOMA})}$.

In Fig. 3, we plot the BER for the 2nd user, with $0.2 \ge \alpha_2 \ge \frac{1}{3}\frac{1}{2} \simeq 0.17 \gg 0.05 \ge \alpha_1$. Note that the

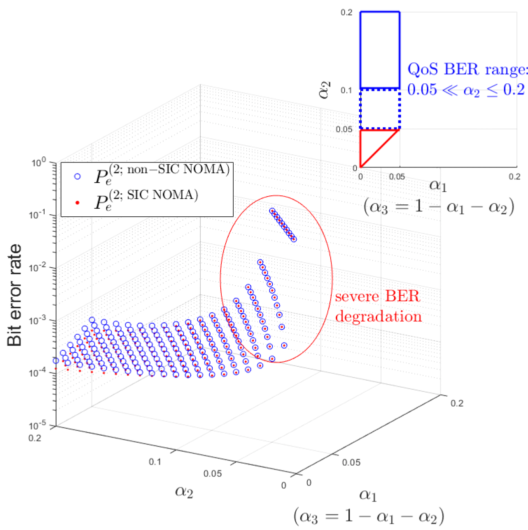

Fig. 3. BER, for QoS and degradation ranges for 2nd user.

3rd user uses $\alpha_3 \geq 0.75$, and then $\alpha_1 + \alpha_2 \leq 0.25$, i.e., the power allocation less than 0.25 remains for the 1st user and the 2nd user. And for the user fairness, $\alpha_2 \gg \alpha_1$ is required. As shown in Fig. 2, the non-SIC NOMA BER is comparable to the standard SIC NOMA BER, with the small BER loss.

In Fig. 4, we plot the BER for the 1st user, with the same ranges in Fig. 2. As shown in Fig. 3, the non-SIC NOMA only suffers the severe BER degradation, for $\alpha_2 < 0.1$. Such range, however, is avoided, because the 2nd user BER degrades severely. Then, the non-SIC NOMA BER is comparable to the standard SIC NOMA BER, with the small BER loss, for $0.1 < \alpha_2 < 0.2$.

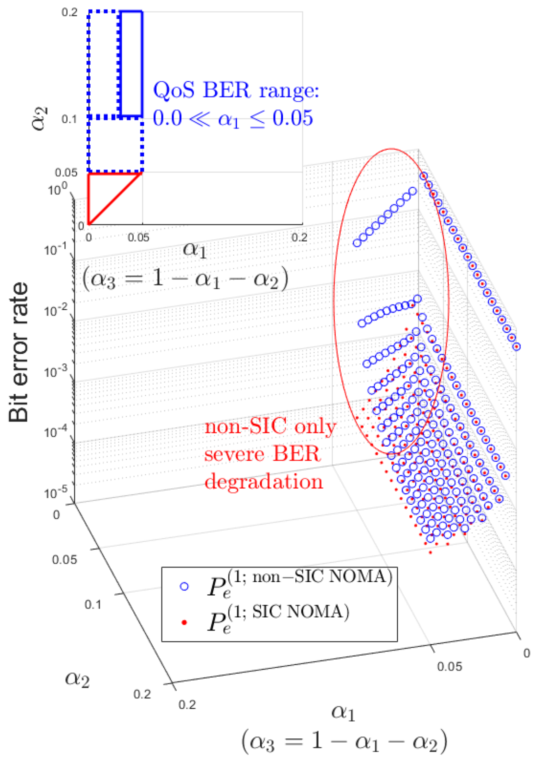

Fig. 4. BER, for QoS and degradation ranges for 1st user.

V. Conclusion

In this letter, we proposed the non-SIC NOMA scheme with M-user. First, we derived the closed-form BER expression. Then it was shown that the BER performance of this non-SIC NOMA scheme is comparable to that of the standard SIC NOMA scheme, with the power allocation of the user fairness, and the optimal ML detection. In result, the implementation of NOMA without SIC complexity could be considered in the practical systems.

Appendix

We validate the BER approximation in this letter, by the numerical results of Section IV. Remark that there are two causes of the approximation; the initial cause is the decision boundary approximation, and the secondary cause is the dominant term approximation. The decision boundary approximation is tolerated by the following observation; for example, the approximate decision boundary $r_2^{(\text{db; 1}st\text{; non-SIC NOMA})} = +|h_2|\sqrt{P}\sqrt{\alpha_3}$ is obtained from the equal likelihood equation

$$p_{R_2|B_2}(r_2 \mid b_2 = 0) = p_{R_2|B_2}(r_2 \mid b_2 = 1), \tag{17}$$

which is given by

$$
\begin{aligned}
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(+\sqrt{\alpha_3}+\sqrt{\alpha_2}+\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}\\
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(+\sqrt{\alpha_3}+\sqrt{\alpha_2}-\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}\\
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(-\sqrt{\alpha_3}+\sqrt{\alpha_2}+\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}\\
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(-\sqrt{\alpha_3}+\sqrt{\alpha_2}-\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}\\
&=\\
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(+\sqrt{\alpha_3}-\sqrt{\alpha_2}+\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}\\
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(+\sqrt{\alpha_3}-\sqrt{\alpha_2}-\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}\\
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(-\sqrt{\alpha_3}-\sqrt{\alpha_2}+\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}\\
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(-\sqrt{\alpha_3}-\sqrt{\alpha_2}-\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}.
\end{aligned}
\tag{18}
$$

Note that, at $r_2^{(\mathrm{db};\,1st;\,\text{non-SIC NOMA})}=+|h_2|\sqrt{P}\sqrt{\alpha_3}$,

$$
\begin{aligned}
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(+\sqrt{\alpha_3}+\sqrt{\alpha_2}+\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}\\
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(+\sqrt{\alpha_3}+\sqrt{\alpha_2}-\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}\\
&=\\
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(+\sqrt{\alpha_3}-\sqrt{\alpha_2}+\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}\\
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(+\sqrt{\alpha_3}-\sqrt{\alpha_2}-\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}.
\end{aligned}
\tag{19}
$$

And the four terms approximated to zero are so small, that we could not obtain the numerical values, for $P/N_0=40\ \mathrm{dB}$ in our case. Instead, for $P/N_0=20\ \mathrm{dB}=100$, we give the numerical values, at $r_2^{(\mathrm{db};\,1st;\,\text{non-SIC NOMA})}=+|h_2|\sqrt{P}\sqrt{\alpha_3}$, with $\alpha_1=0.05$, $\alpha_2=0.2$, and $|h_2|^2=1.0$, as follows

$$
\begin{aligned}
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(-\sqrt{\alpha_3}+\sqrt{\alpha_2}+\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}\simeq 1.2251\times 10^{-50},\\
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(-\sqrt{\alpha_3}+\sqrt{\alpha_2}-\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}\simeq 1.5112\times 10^{-100},\\
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(-\sqrt{\alpha_3}-\sqrt{\alpha_2}+\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}\simeq 7.9200\times 10^{-168},\\
&+\frac{1}{4}\frac{1}{\sqrt{2\pi N_0/2}}e^{-\frac{\left(r_2-|h_2|\sqrt{P}\left(-\sqrt{\alpha_3}-\sqrt{\alpha_2}-\sqrt{\alpha_1}\right)\right)^2}{2N_0/2}}\simeq 1.7633\times 10^{-252}.
\end{aligned}
\tag{20}
$$

Therefore, the decision boundary approximation is tolerable. Next, we consider the dominant term approximation. Based on the approximated decision boundaries, the exact BER is given by

$$
\begin{aligned}
&P_e^{(2;\,\text{non-SIC NOMA};\,exact)}=\\
&+P_e^{(2;\,\text{non-SIC NOMA})}\\
&+\underbrace{\frac{1}{4}f\left(+2;+1;+1\right)+\frac{1}{4}f\left(+2;+1;-1\right)}_{3rd\ nearest\ boundary,\ \text{approximated in this letter}}\\
&-\frac{1}{4}f\left(+1;+1;+1\right)-\frac{1}{4}f\left(+1;+1;-1\right)\\
&-\frac{1}{4}f\left(+2;-1;-1\right)-\frac{1}{4}f\left(+2;-1;+1\right),
\end{aligned}
\tag{21}
$$

where

$$
\begin{aligned}
&P_e^{(2;\,\text{non-SIC NOMA})}=\\
&\overbrace{+\frac{1}{2}f\left(+0;+1;+1\right)+\frac{1}{2}f\left(+0;+1;-1\right)}^{1st\ nearest\ boundary}\\
&\underbrace{+\frac{1}{4}f\left(+1;-1;-1\right)+\frac{1}{4}f\left(+1;-1;+1\right)}_{2nd\ to\ (M-m+1)th\ nearest\ boundary},
\end{aligned}
\tag{22}
$$

and

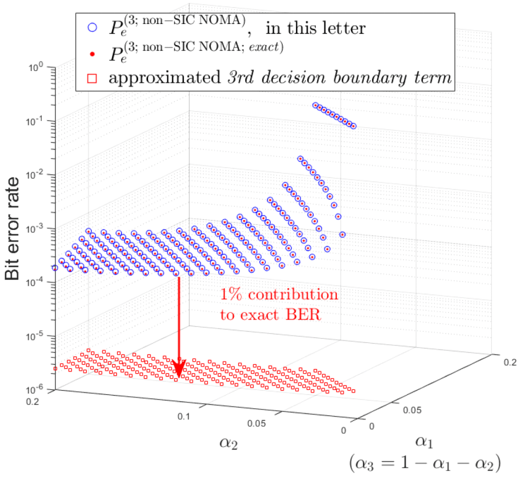

Fig. 5. Comparison of exact BER, BER in this letter, and approximated 3rd decision boundary term, for 2nd user.

$$f\left(C;B;A\right) = F\left(\frac{\Sigma_2 P\left(C\sqrt{\alpha_3} + B\sqrt{\alpha_2} + A\sqrt{\alpha_1}\right)^2}{N_0}\right). \tag{23}$$

In Fig. 5, we compare $P_e^{(2;\,\text{non-SIC NOMA};\,exact)}$, $P_e^{(2;\,\text{non-SIC NOMA})}$, and the $3rd$ nearest decision boundary term. As shown in Fig. 5, the contribution of the $3rd$ nearest decision boundary term to $P_e^{(2;\,\text{non-SIC NOMA};\,exact)}$ is about 1%. Based on the above observations, we approximate the BER as dominant terms of the $2nd$ to $(M-m+1)th$ nearest boundary, in the equation.

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