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On Sufficient Condition for Power Allocation Range of BER for M- User NOMA

Kyuhyuk Chung[◆]

ABSTRACT

Recently, the bit-error rate (BER) for non-orthogonal multiple access (NOMA) with M- user has been derived. However, the ranges of the power allocation coefficients, for which the BER expression holds, are provided as a necessary condition. In this letter, the authors derive a sufficient condition for the power allocation ranges for the BER expression. Such ranges are significant, for each user to avoid the severe performance degradation. In result, the user fairness could be established with the power allocation within the derived ranges.

Key Words : NOMA, user fairness, superposition coding, SIC, power allocation

Ⅰ. Introduction

As a promising multiple access scheme in the fifth generation (5G) mobile networks, non-orthogonal multiple access (NOMA) has received considerable attention [1-6]. Recently, for M- user, the bit-error rate (BER) for NOMA has been derived[7]. On the other hand, in [8], the BER expression was presented with randomly generated signals. In [9], the exact BER expression was derived for the two and three-user cases. The exact average symbol error rate (SER) expressions for the two-user case were presented in [10].

In this letter, we derive a sufficient condition of the power allocation ranges, for which the BER expression holds. And we show that the sufficient range is significant, in order for each user to avoid the severe BER degradation.

The main assumption for the BER expression is the nonlinear inequality, which involves the square-root. Therefore, a linear inequality was presented in [7]. However, such inequality is a necessary condition, for which the BER expression does not always hold. Thus, in this letter, we propose the tight sufficient condition, for which the BER expression always holds.

Ⅱ. System and Channel Model

We consider a cellular downlink NOMA. A base station and M users are within the cell. The Rayleigh fading channel between the mth user and the base station is denoted by $h_m \sim \mathcal{CN}(0,\Sigma_m)$, $1 \le m \le M$. The base station will send the superimposed signal $x=\sum_{m=1}^{M}\sqrt{\alpha_m P}s_m$, where s_m is the message for the mth user, α_m is the power allocation coefficient, with $\sum_{m=1}^{M}\alpha_m=1$, $\alpha_1 \le \cdots \le \alpha_M$, and P is the constant total transmitted power at the base station. The power of s_m is normalized as unit power, $\mathbb{E}[s_m s_m^*]=\mathbb{E}[|s_m|^2]=1$. The observation at the mth user is given by

$$y_m = h_m x + w_m, \tag{1}$$

where $w_m \sim \mathcal{CN}(0,N_0)$ is additive white Gaussian noise (AWGN). We consider the binary phase shift keying (BPSK) modulation, with $s_m \in \{+1,-1\}$. Then the following metric is the sufficient statistics

$$r_m = |h_m| x + n_m, \tag{2}$$

where $n_m \sim \mathcal{N}(0,N_0/2)$. Let the information input bit for the mth user be $b_m \in \{0,1\}$. Then, the

◆ First Author : (ORCID:0000-0001-5429-2254)Department of Software Science, Dankook University, khchung@dankook.ac.kr, 교수, 종신회원
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bit-to-symbol mapping is given as

$$\begin{cases} s_m(b_m=0)=+1 \\ s_m(b_m=1)=-1 \end{cases}, \text{ or } s_m(b_m)=(-1)^{b_m}. \quad (3)$$

And we use the binary representation notation for the index i, $0 \le i \le 2^{m-1}-1$,

$$(i)_2 = b_{m-1}b_{m-2}\cdots b_2b_1, \quad (4)$$

where each bit b_j, $1 \le j \le m-1$, corresponds to the jth user. In [7], the BER performance of N-user NOMA with the perfect successive interference cancellation (SIC) and maximum likelihood (ML) detection is derived, when the channels are sorted by the instantaneous realizations, i.e., $|h_1| \ge \cdots \ge |h_M|$.

In this letter, we derive the BER expression, under the sort by the channel gain variances, i.e., $\Sigma_1 \ge \cdots \ge \Sigma_M$. The derivation of the BER expression in [7] is nontrivial; we review it here to motivate our proposed sufficient ranges in this letter. First, the likelihood for the information bit b_m for the mth user is expressed as

$$p_{R_m|B_m}(r_m \mid b_m) = \frac{1}{2^{m-1}}\frac{1}{\sqrt{2\pi N_0/2}}\sum_{i=0}^{2^{m-1}-1} e^{-\frac{\left(r_m-(-1)^{b_m}|h_m|\sqrt{P}\sqrt{\alpha_{m,i}}\right)^2}{2N_0/2}}, \quad (5)$$

where $\alpha_{m,i}$ is given by

$$\alpha_{m,i} = \left(\sqrt{\alpha_m} + (-1)^{b_{m-1}}\sqrt{\alpha_{m-1}} + \ldots + (-1)^{b_1}\sqrt{\alpha_1}\right)^2. \quad (6)$$

Note that the subscript i of the above expression is related with the summation index i in the equation. The optimum detection is made, based on the maximum likelihood (ML), as

$$\hat{b}_m = \arg\max_{b_m \in \{0,1\}} p_{R_m|B_m}(r_m \mid b_m). \quad (7)$$

Here, we give the main assumption, which is significant throughout this letter

$$\sqrt{\alpha_m} \ge \sum_{i=1}^{m-1}\sqrt{\alpha_i}, \quad \text{for } 2 \le m \le M. \quad (8)$$

If the above condition is satisfied, then the one exact decision boundary is

$$r_m = 0. \quad (9)$$

Then, the decision regions are given by

$$\begin{cases} b_m = 0: & 0 < r_m \\ b_m = 1: & r_m < 0. \end{cases} \quad (10)$$

Therefore, the conditional BER of the mth user can be expressed by

$$P_{e\|h_m|}^{(m;\, ideal\ perfect\ SIC + ML\ optimal)} = \frac{1}{2^{m-1}}\sum_{i=0}^{2^{m-1}-1} Q\left(\sqrt{\frac{|h_m|^2 P\alpha_{m,i}}{N_0/2}}\right), \quad (11)$$

where $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\, dz$. Note that up to now, the BER derivations of both the channel sort models for $|h_1| \ge \cdots \ge |h_M|$ and $\Sigma_1 \ge \cdots \ge \Sigma_M$ are exactly the same. If the channel gains are sorted by $|h_1| \ge \cdots \ge |h_M|$, the BER expression is presented in [7]. On the other hand, if the channel gains are sorted by $\Sigma_1 \ge \cdots \ge \Sigma_M$, then we can use the well-known Rayleigh fading integration formula,

$$\int_0^\infty Q\left(\sqrt{2\gamma}\right)\frac{1}{\gamma_b}e^{-\frac{\gamma}{\gamma_b}}\, d\gamma = \frac{1}{2}\left(1-\sqrt{\frac{\gamma_b}{1+\gamma_b}}\right), \quad (12)$$

where the random variable (RV) γ is exponentially distributed

$$\gamma = \frac{|h_m|^2 P\alpha_{m,i}}{N_0}, \tag{13}$$

and the mean of γ is defined as

$$\begin{aligned}\gamma_b = \mathbb{E}[\gamma] = \mathbb{E}\left[\frac{|h_m|^2 P\alpha_{m,i}}{N_0}\right] \\ = \frac{\Sigma_m P\alpha_{m,i}}{N_0}.\end{aligned} \tag{14}$$

Therefore, the BER of the mth user can be expressed by

$$P_e^{(m;\, ideal\ perfect\ SIC + ML\ optimal)} = \frac{1}{2^{m-1}} \sum_{i=0}^{2^{m-1}-1} F\left(\frac{\Sigma_m P\alpha_{m,i}}{N_0}\right), \tag{15}$$

where for the compact presentation of Rayleigh fading BER performance, we use the notation as

$$F(\gamma_b) = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_b}{1+\gamma_b}}\right). \tag{16}$$

Note that γ_b is defined in the equation .

Ⅲ. Sufficient Condition Derivation for Power Allocation Range of BER Expression

The main assumption in for the BER expression is the nonlinear inequality, i.e., $\sqrt{\alpha_m} \geq \sum_{i=1}^{m-1} \sqrt{\alpha_i}$, for $2 \leq m \leq M$, because it involves the square-root. Therefore, a linear inequality was presented in [7]

$$\begin{cases} \alpha_M \geq \frac{1}{2} \\ \alpha_m \geq \frac{1}{2} - \sum_{i=m+1}^{M} \frac{1}{2}\alpha_i, \quad \text{for } m < M \end{cases}. \tag{17}$$

The above necessary ranges are intuitive, because from the power allocation $\alpha_M \geq \frac{1}{2}$ of the weakest channel user, the power allocation α_m of the stronger channel user is obtained by recursively reducing the power allocation of the weaker channel users in half, i.e., $\sum_{i=m+1}^{M} \frac{1}{2}\alpha_i$. However, for some values in the necessary ranges, the BER expression in does not hold. For example, for $M = 3$, with $\alpha_3 = 6/10$, $\alpha_2 = 3/10$, and $\alpha_1 = 1/10$,

$$\sqrt{\alpha_3} < \sqrt{\alpha_2} + \sqrt{\alpha_1}, \tag{18}$$

which does not satisfy the main assumption in for the BER expression in the equation . Therefore, we derive the tight sufficient ranges for the BER expression in the equation . We start the $m = M$ case from the main assumption of the inequality , and then obtain the recursive inequality. We observe that the RHS of the inequality is the maximum, when $\alpha_{M-1} = \alpha_{M-2} = \cdots = \alpha_1$. In this case, the inequality can be expressed as

$$\alpha_M \geq (M-1)^2 \alpha_{M-1}. \tag{19}$$

Using $\sum_{m=1}^{M} \alpha_m = 1$, we obtain

$$\alpha_M \geq \frac{M-1}{M}. \tag{20}$$

Now, we obtain the recursive inequality. Similarly, we observe that the RHS of the inequality is the maximum, when $\alpha_{m-1} = \alpha_{m-2} = \cdots = \alpha_1$, for the m case . In this case, the inequality can be expressed as

$$\alpha_m \geq (m-1)^2 \alpha_{m-1}. \tag{21}$$

Using $\sum_{m=1}^{M} \alpha_m = 1$, we obtain

$$\alpha_m \geq \frac{m-1}{m}\left(1-\sum_{j=m+1}^{M}\alpha_j\right),\quad \text{for } 2 \leq m \leq M-1. \quad (22)$$

Lastly, $m=1$, we obtain simply $\alpha_1 = 1-\sum_{m=2}^{M}\alpha_m$.

Ⅳ. Results and Discussions

We consider the 3rd user within the cell with $M=3$. Assume $\Sigma_1 = 1.5$, $\Sigma_2 = 1.0$, and $\Sigma_3 = 0.5$, and the constant total transmitted signal power to noise power ratio $P/N_0 = 40$ dB.

First, in Fig. 1, we depict the BER of the 3rd user for the sufficient range in , i.e., $\alpha_3 \geq \frac{2}{3}$. For this range, the BER expression in holds. However, outside the sufficient range, i.e., $\alpha_3 < \frac{2}{3}$, the BER expression in dose not hold, generally. In Fig. 2, we depict the BER, for outside the sufficient range. As shown in Fig. 2, the sufficient range boundary is tight to the exact range, while the necessary range boundary is loose to the exact range. (For the better presentation, we plot the figures without $\alpha_2 \geq \alpha_1$.)

Note that the boundary of the exact range starts at $\alpha_2 = \alpha_1 = \frac{1}{3}\times\frac{1}{2}$. However, the boundary of the exact range is not a line, i.e., $\alpha_2 + \alpha_1 = \frac{1}{3}$. Rather, the boundary of the exact range is the part of an ellipse-like shape, i.e.,

$$\alpha_2 = \left(\frac{-2\sqrt{\alpha_1}+\sqrt{8-12\alpha_1}}{4}\right)^2, \quad (23)$$

which is between the necessary range boundary and the proposed sufficient range boundary, in Fig. 2.

Remark that the BER performance suffers the severe BER degradation, at the near to the exact range boundaries. Therefore, the power allocations should be avoided near the exact range boundaries. We observe that the BER at the boundary of the proposed sufficient range is much lower than the BER at the boundary of the exact range, except the vicinity of $(\alpha_1, \alpha_2) = \left(\frac{1}{3}\times\frac{1}{2}, \frac{1}{3}\times\frac{1}{2}\right)$. Lastly, we should mention that the proposed sufficient ranges hold for any M, and the reason why we showed an example of $M=3$ is to have the better intuition and perspective.

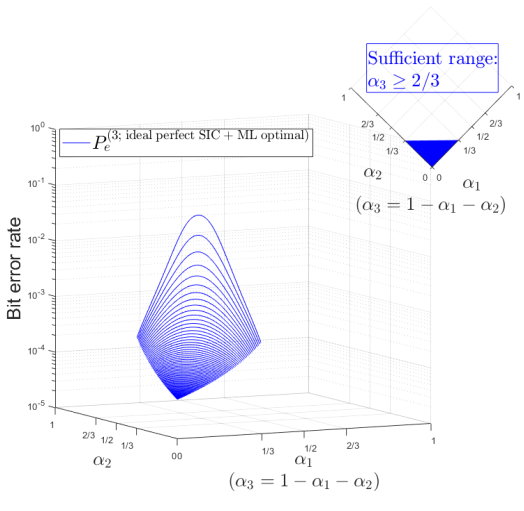

Fig. 1. BER and sufficient range for 3rd user.

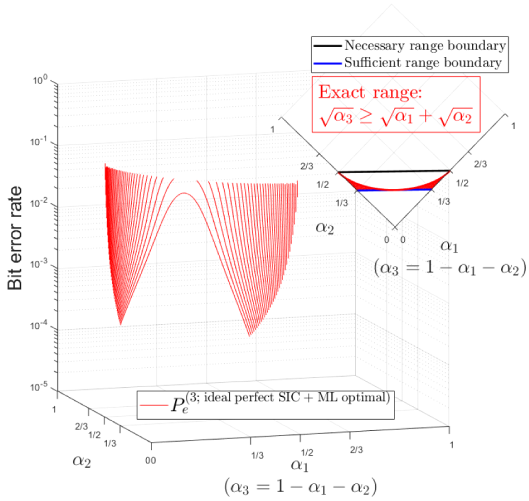

Fig. 2. BER, necessary range boundary, and exact range for 3rd user.

V. Conclusion

In this letter, we derived a sufficient condition of the power allocation ranges for BER of NOMA with M- user. We showed that the derived sufficient ranges are significant for two reasons; one is that obviously the BER expression should hold for the given range, and the other is that the severe BER degradation of each user could be avoided with the proposed ranges.

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