논문 20-45-10-06 The Journal of Korean Institute of Communications and Information Sciences '20-10 Vol.45 No.10
https://doi.org/10.7840/kics.2020.45.10.1694

On BER for M-User NOMA with Symmetric SC

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ABSTRACT

In non-orthogonal multiple access (NOMA), the ideal perfect successive interference cancellation (SIC) is validated theoretically by the random channel coding, practically such as low-density parity-check (LDPC) codes or turbo codes. Recently, symmetric superposition coding (SSC) has been shown to achieve the ideal perfect SIC bit-error rate (BER) performance by simulations with as many as three users, while the authors of this letter derived the analytical expression for BER, only for to a two-user scenario. In this letter, the derivation of the closed-form BER expression for M users is reported. It is shown that the SSC NOMA achieves near-perfect SIC BER performance. This is possible for with two reasons; one is the power allocation of the user fairness, and the other is the optimal maximum-likelihood (ML) detection.

Key Words : NOMA, user fairness, superposition coding, SIC, power allocation

Ⅰ. Introduction

For the fifth generation (5G) mobile networks, non-orthogonal multiple access (NOMA) has been considered a promising multiple access scheme[1-2]. Recently, symmetric superposition coding (SC) has been proposed to achieve the ideal perfect successive interference cancellation (SIC) bit-error rate (BER) performance[3]. In this letter, the derivation of the analytical expression for the BER of symmetric SC (SSC) NOMA with M users , is reported, and the BER of SSC NOMA is compared with to that of the ideal perfect SIC NOMA.

Compared to the previous works, we summarize our contributions in this letter; first, SSC was proposed, and the performance was verified by simulations, in [3]. And for two-user SSC NOMA, the analytical expression for the BER was presented in [4], and for M-user standard SC NOMA, the BER expression was derived in [5]. Then, in this letter, we derive the closed-form expression for M-user SSC NOMA, and verify the BER expression by numerical results, compared to both the ideal perfect SIC NOMA and the non-SIC NOMA.

Ⅱ. System and Channel Model

A cellular downlink NOMA transmission system with a base station and users within the cell is considered. The Rayleigh fading channel between the mth user and the base station is denoted by $h_m \sim \mathcal{CN}(0,\Sigma_m)$, $1 \le m \le M$. The base station sends the superimposed signal $x=\sum_{m=1}^{M}\sqrt{\alpha_m P}s_m$, where s_m is the message for the mth user, α_m is the power allocation coefficient, with $\sum_{m=1}^{M}\alpha_m=1$, and P is the total transmitted power at the base station. The power of s_m is normalized as unit power, $\mathbb{E}[s_m s_m^*]=\mathbb{E}[|s_m|^2]=1$. The observation at the mth user is given by

$$r_m=|h_m|x+n_m \tag{1}$$

where $n_m \sim \mathcal{N}(0,N_0/2)$ is additive white Gaussian noise (AWGN). Binary phase shift keying (BPSK) modulation with $s_m \in \{+1,-1\}$ is considered. For the information input message index i, $0 \le i \le 2^M-1$, the binary representation notation,

$$(i)_2=b_M b_{M-1} b_{M-2} \cdots b_2 b_1 \tag{2}$$

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논문번호 : 202005-127-A-LU, Received June 13, 2020; Revised July 4, 2020; Accepted July 13, 2020

and the binary Gray code representation,

$$(i)_2^{Gray} = g_M g_{M-1} g_{M-2} \cdots g_2 g_1 \tag{3}$$

are used, where each bits b_j and g_j, $1 \le j \le M$, correspond to the jth user. Then, the bit-to-symbol mapping is given as, for the standard NOMA,

$$\begin{cases} s_m(b_m = 0) = +1 \\ s_m(b_m = 1) = -1 \end{cases}, \text{ or } s_m(b_m) = (-1)^{b_m} \tag{4}$$

and for SSC NOMA,

$$\begin{cases} s_m(g_m = 0) = +1 \\ s_m(g_m = 1) = -1 \end{cases}, \text{ or } s_m(g_m) = (-1)^{g_m}. \tag{5}$$

In a recent study[5], the BER performance of M-user NOMA with ideal perfect SIC and maximum likelihood (ML) detection was derived, when the channels are sorted by the instantaneous realizations, that is, $|h_1| \ge \cdots \ge |h_M|$. In this letter, under the sort by the channel gain variances, i.e., $\Sigma_1 \ge \cdots \ge \Sigma_M$, the BER of the mth user can be expressed by

$$P_e^{(m;\,\text{ideal perfect SIC NOMA})} = \sum_{i=0}^{2^{m-1}-1} \frac{1}{2^{m-1}} F\left(\frac{\Sigma_m P \alpha_{m,i}}{N_0}\right) \tag{6}$$

where $\sqrt{\alpha_m} \ge \sum_{j=1}^{m-1} \sqrt{\alpha_j}$, $2 \le m \le M$. In addition, $\alpha_{m,i}$ is given by

$$\alpha_{m,i} = \left(\sqrt{\alpha_m} + (-1)^{b_{m-1}} \sqrt{\alpha_{m-1}} + \ldots + (-1)^{b_1} \sqrt{\alpha_1}\right)^2 \tag{7}$$

For the compact presentation of Rayleigh fading BER performance, the notation

$$F(\gamma_b) = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_b}{1+\gamma_b}}\right). \tag{8}$$

is used. Furthermore, the compact presentation of BER performance, the notation

$$P_e^{(m;\,\text{ideal perfect SIC NOMA})}(I;C;\alpha) = \sum_{i=I}^{2^{m-1}-1} \frac{1}{2^{m-1}} F\left(\frac{\Sigma_C P \alpha}{N_0}\right) \tag{9}$$

is used, where I is the lower limit of the summation index, C is the channel index, and α is the effective power allocation. With the above notation,

$$\begin{aligned} &P_e^{(m;\,\text{ideal perfect SIC NOMA})} = \\ &P_e^{(m;\,\text{ideal perfect SIC NOMA})}(0;m;\alpha_{m,i}). \end{aligned} \tag{10}$$

Ⅲ. BER Derivation for SSC M-user NOMA

First, for $m = M$, the first nearest decision boundary is $r_M = 0$. Then, the SSC NOMA BER of the Mth user equals the ideal perfect SIC NOMA BER

$$P_e^{(M;\,\text{SSC NOMA})} = P_e^{(M;\,\text{ideal perfect SIC NOMA})} \tag{11}$$

because $g_M = b_M$. For $m = M-1$, the first nearest decision boundary $r_M = 0$ for

Table 1. Axis examples ($1 = \alpha_3 + \alpha_2 + \alpha_1$)

α_3 $(= 1 - \alpha_1 - \alpha_2)$ (hidden-axis)	α_1 (x-axis)	α_2 (y-axis)
1	0	0
0.7	0	0.3
0.7	0.05	0.25
0.7	0.3	0
0	1	0
0	0	1
1 / 3	1 / 3	1 / 3
0.9	0	0.1
0.9	0.1	0

$P_e^{(M;\,\text{SSC NOMA})}$ becomes the second nearest decision boundary $r_{M-1}=0$ for $P_e^{(M-1;\,\text{SSC NOMA})}$, and the two first nearest decision boundaries $r_{M-1}=\pm|h_{M-1}|\sqrt{P\alpha_M}$ for $P_e^{(M-1;\,\text{SSC NOMA})}$ correspond to the first nearest decision boundary $r_{M-1}=0$ for $P_e^{(M-1;\,\text{ideal perfect SIC NOMA})}$. Then, the SSC NOMA BER of the $(M-1)th$ user is given by

$$P_e^{(M-1;\,\text{SSC NOMA})} \simeq \overbrace{P_e^{(M-1;\,\text{ideal perfect SIC NOMA})}}^{1st\ nearest\ boundary} + \underbrace{P_e^{(M;\,\text{ideal perfect SIC NOMA})}(I;C;\alpha)}_{2nd\ nearest\ boundary} \tag{12}$$

where $I=2^{M-1}-2^{M-2}$, $C=M-1$, and $\alpha=\left(\sqrt{\alpha_{M,i}}+\sqrt{\alpha_M}\right)^2$. Similarly, for $1\le m\le M-1$, the SSC NOMA BER of the mth user is derived as

$$P_e^{(m;\,\text{SSC NOMA})} \simeq \overbrace{P_e^{(m;\,\text{ideal perfect SIC NOMA})}}^{1st\ nearest\ boundary} + \underbrace{\sum_{j=m+1}^{M} P_e^{(j;\,\text{deal perfect SIC NOMA})}(I;C;\alpha)}_{2nd\ to\ (M-m+1)th\ nearest\ boundaries} \tag{13}$$

where $I=2^{j-1}-2^{m-1}$, $C=m$, and $\alpha=\left(\sqrt{\alpha_{j,i}}+\sqrt{\alpha_j}\right)^2$. The approximation up to the $(M-m+1)th$ nearest decision boundary is tolerable because, from the $(M-m+2)th$ nearest decision boundary, the contribution to BER is small.

Ⅳ. Results and Discussions

All three users within the cell are considered with $M=3$. It is assumed that $\Sigma_1=1.5$, $\Sigma_2=1.0$, and $\Sigma_3=0.5$. From such channel gain variances, we can consider the distance parameter as follows[6];

$$h_i:h_j=\frac{g_i}{\sqrt{1+d_i^\beta}}:\frac{g_j}{\sqrt{1+d_j^\beta}}, \tag{14}$$

where g_i and $g_j \sim \mathcal{CN}(0,1^2)$ denote the unit variance Rayleigh fading channel gain, β is the path loss factor, and d_i and d_j denotes the distance from the ith and jth user to the base station. Then we derive the following equation

$$\frac{\mathbb{E}[|h_i|^2]}{\mathbb{E}[|h_j|^2]}=\frac{\Sigma_i}{\Sigma_j}=\frac{\mathbb{E}\left[\left|\frac{g_i}{\sqrt{1+d_i^\beta}}\right|^2\right]}{\mathbb{E}\left[\left|\frac{g_j}{\sqrt{1+d_j^\beta}}\right|^2\right]}=\frac{\frac{1}{1+d_i^\beta}}{\frac{1}{1+d_j^\beta}},$$

$$\frac{1+d_j^\beta}{1+d_i^\beta}=\frac{\Sigma_i}{\Sigma_j},$$

$$d_j^\beta=\frac{\Sigma_i}{\Sigma_j}-1+\frac{\Sigma_i}{\Sigma_j}d_i^\beta. \tag{15}$$

From the above equation, for example, with $\beta=4$, and $d_1=10\text{ cm}$, we have $d_2\simeq 84\text{ cm}$ and $d_3\simeq 119\text{ cm}$.

Before we present the figures, in order to avoid confusion about x axis and y axis, we explain the axes in detail as follows; we note that

$$1=\alpha_1+\alpha_2+\alpha_3. \tag{16}$$

Then α_1, α_2, and α_3 are dependent on each other. Therefore, with the two power allocation coefficients, we can represent the other. Thus, we choose α_1 and α_2. Then α_2 can be represented as

$$\alpha_3=1-\alpha_1-\alpha_2. \tag{17}$$

For the better understanding, we give the numerical relationship among α_1, α_2, and α_3, in Table 1.

First, we consider the constant total transmitted signal power to noise power ratio (SNR) $P / N_0 = 40$ dB.

In Fig. 1, the BER for the third user is plotted, with $\alpha_3 \geq \frac{2}{3} \simeq 0.67$. To avoid severe BER degradation, more power is required, i.e., $\alpha_3 \geq 0.7$. As shown in Fig. 1, the BER performance of the SSC NOMA is the same as that of the ideal perfect SIC NOMA because $g_M = b_M$.

In Fig. 2, the BER for the second user is plotted, with $\alpha_2 \leq 0.25$. For user fairness, $\alpha_2 \gg \alpha_1$ is required. As shown in Fig. 2, the SSC NOMA BER is close to the ideal perfect SIC NOMA BER, with the small BER loss.

In Fig. 3, the BER for the first user, with the same ranges shown in Fig. 2. The quality of service (QoS) is provided with $0 \ll \alpha_1 \leq 0.05$. As shown in Fig. 3, the SSC NOMA BER is also close to the ideal perfect SIC NOMA BER, with the small BER loss.

Next, we also depict the three users' BERs versus

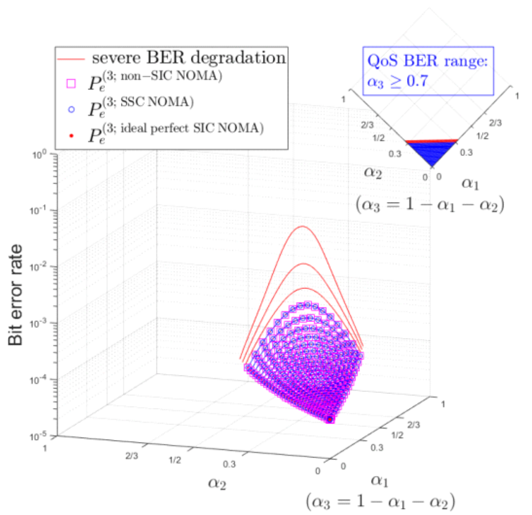

Fig. 1. BER, for QoS and degradation ranges of 3rd user.

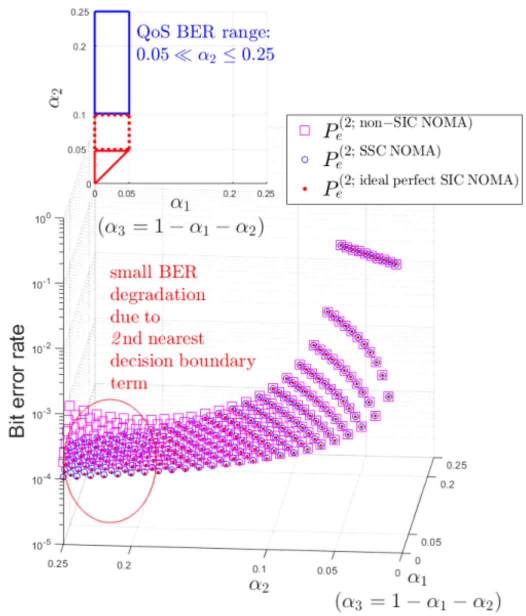

Fig. 2. BER, for QoS and degradation ranges of 2*nd* user.

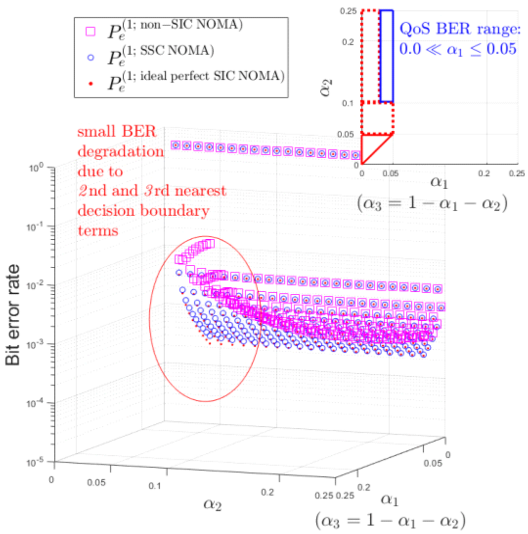

Fig. 3. BER, for QoS and degradation ranges of 1*st* user.

the SNR P / N_0, to verify the proposed scheme, in Fig. 4 and Fig. 5, with $\{\alpha_1, \alpha_2, \alpha_3\} = \{0.05, 0.25, 0.7\}$ and $\{0.08, 0.2, 0.72\}$, respectively. As shown in Fig. 4 and Fig. 5, the SSC NOMA BER is close to the ideal perfect SIC NOMA BER, with the small BER loss, for each

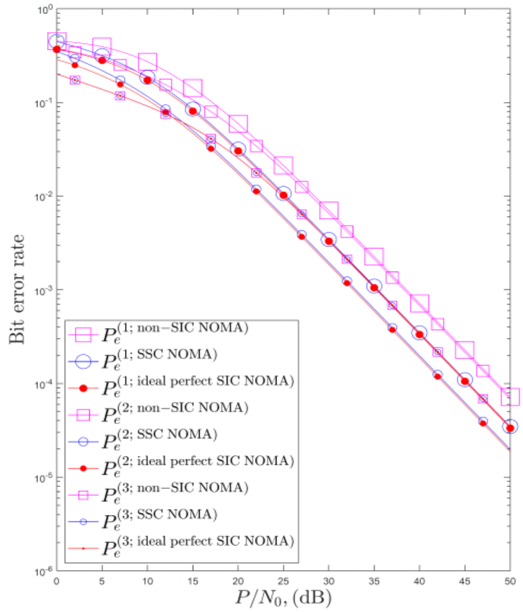

Fig. 4. BER versus P/N_0, for all three users.

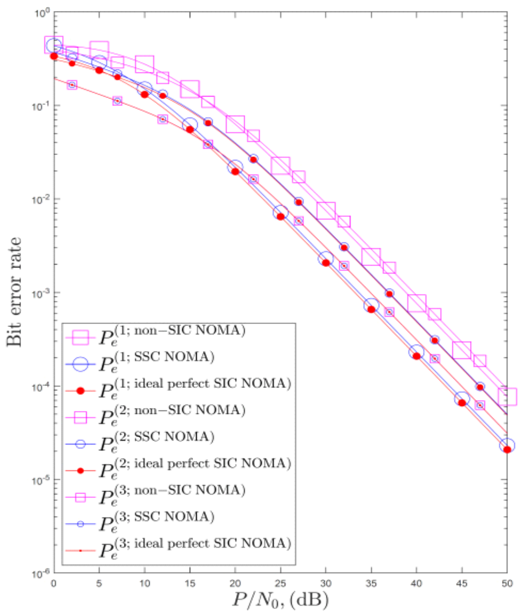

Fig. 5. BER versus P/N_0, for all three users.

user.

In addition, in order to verify the proposed scheme, we also include the non-SIC NOMA scheme[7,8], in Fig. 1, 2, 3, and 4. Note that the BER of the ideal perfect SIC NOMA scheme is the ultimate upper bound, while the BER $P_e^{(m;\,\text{non-SIC NOMA})}$ of the non-SIC NOMA scheme is the lower bound, for the proposed scheme, respectively,

$$\underbrace{P_e^{(m;\,\text{ideal perfect SIC NOMA})}}_{\text{ultimate lower bound}} \le P_e^{(m;\,\text{SSC NOMA})} \le \underbrace{P_e^{(m;\,\text{non-SIC NOMA})}}_{\text{upper bound}}. \tag{18}$$

As shown in Fig. 1, 2, 3, and 4, the SSC NOMA BER is better than the non-SIC NOMA BER.

Lastly, the BER expression is dependent on Σ_m and $\alpha_{m,i}$. Therefore, each user's BER performance is different, according to the channel gain variance Σ_m and the power allocation $\alpha_{m,i}$.

V. Conclusion

The analytical expression for the BER of the SSC NOMA scheme with M users was derived. It was shown that the SSC NOMA scheme with three users achieves near-perfect SIC BER performance, with the power allocation of the user fairness, and the optimal ML detection.

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