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Additional Condition on CSCG Random Vectors for I/Q Channels: With Application to CIS in NOMA

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ABSTRACT

Circularly-symmetric complex Gaussian (CSCG) random vectors (Rvecs) are often used to model practical in-phase and quadrature (I/Q) channels in communication systems. However, we know that CSCG Rvecs do not completely encompass practical I/Q channels. Thus, in this study, we propose an additional condition for CSCG Rvecs to be valid over I/Q channels, namely, a purely-real complex correlation coefficient (CCC). We then investigate the effects of an invalid CCC on the achievable data rate of non-orthogonal multiple access (NOMA) with correlated information sources. Results revealed that a purely-real CCC constraint should be imposed on CSCG Rvecs, when considering practical I/Q channels.

Key Words : CSCG random vector, in-phase/quadrature channels, NOMA, power allocation, correlation coefficient.

Ⅰ. Introduction

One of the big differences between non-orthogonal multiple access (NOMA) and orthogonal multiple access (OMA) is channel sharing[1-6]. This difference opens the possibility for broadcasting common information simultaneously in NOMA, unlike in OMA. Recently, the achievable rate region for NOMA with correlated information sources (CIS) was investigated, and it was determined that the achievable rate region of NOMA with CIS is greatly dependent on complex correlation coefficients (CCCs)[7]. However, the exact condition for CCCs over practical in-phase and quadrature (I/Q) channels is unknown. Therefore, in this study, we derive this exact condition, which is validated by numerical results.

Ⅱ. System and Channel Model

In a downlink NOMA system, all users are assumed to experience block fading in a narrow band. The base station and M users are located within the cell. The complex channel coefficient between the mth user and the base station is denoted by h_m. The channels are sorted as $|h_1| \geq |h_2| \geq \cdots \geq |h_m|$. The base station transmits the superimposed signal $x=\sum_{m=1}^{M}\sqrt{\beta_m P_A}c_m$, where c_m is the message for the mth user, β_m is the power allocation coefficient for CIS (with $\sum_{m=1}^{M}\beta_m=1$), and P_A is the total allocated power. The power of the message c_m for the mth user is normalized to unit power, denoted by $\rho_{m,m}=\mathbb{E}\left[|c_m|^2\right]=1$, $\forall m,\ 1\leq m\leq M$. The CCC between the ith and jth users is denoted by $\rho_{i,j}=\mathbb{E}\left[c_i c_j^*\right]$, $\forall i,j,\ i\neq j,\ 1\leq i,j\leq M$. Thus, given the constant total transmitted power P at the base station, P_A is effectively scaled down to

$$P_A\sum_{i=1}^{M}\sum_{j=1}^{M}\rho_{i,j}\sqrt{\beta_i\beta_j}=P. \tag{1}$$

It should be noted that P_A is real because P is real and $\rho_{i,j}+\rho_{j,i}=2\mathrm{Re}\{\rho_{i,j}\}$. The observation at the mth user is given by

$$y_m=h_m x+n_m, \tag{2}$$

where $n_m\sim CN(0,\sigma^2)$ is the additive white Gaussian noise at the mth user.

For two-user NOMA, it should be noted that when successive interference cancellation (SIC) is

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performed, the achievable data rate $R_1^{(\text{SIC; CIS})}$ of the first user is expressed as [7]

$$R_1^{(\text{SIC; CIS})} = \log_2\left(1 + \frac{|h_1|^2 P_A \beta_1 \left(1 - |\rho_{1,2}|^2\right)}{\sigma^2}\right). \quad (3)$$

The achievable data rate $R_1^{(\text{SIC; CIS})}$ is defined by conditional mutual information $I(y_1; c_1 \mid c_2) = h(y_1 \mid c_2) - h(y_1 \mid c_1, c_2)$, where $h(y_1 \mid c_2)$ and $h(y_1 \mid c_1, c_2)$ are the conditional differential entropies[8].

Ⅲ. Brief Review of CSCG Rvec

Let C be a random complex vector of size M for the CIS signals of M users

$$C = \left[\sqrt{P_A\beta_1}c_1, \sqrt{P_A\beta_2}c_2, \cdots, \sqrt{P_A\beta_M}c_M\right]^{\mathrm{T}}, \quad (4)$$

and let the corresponding real random vector D of size $2M$ consist of real and imaginary components of C taken in the order

$$D = \begin{bmatrix} \mathrm{Re}\{\sqrt{P_A\beta_1}c_1\} \\ \mathrm{Re}\{\sqrt{P_A\beta_2}c_2\} \\ \vdots \\ \mathrm{Re}\{\sqrt{P_A\beta_M}c_M\} \\ \mathrm{Im}\{\sqrt{P_A\beta_1}c_1\} \\ \mathrm{Im}\{\sqrt{P_A\beta_2}c_2\} \\ \vdots \\ \mathrm{Im}\{\sqrt{P_A\beta_M}c_M\} \end{bmatrix}. \quad (5)$$

Then, the covariance matrix K_C of the jointly-Gaussian complex random M-vector C is given by

$$\mathrm{K}_C = \mathbb{E}\left[CC^{\mathrm{H}}\right], \quad (6)$$

and the pseudo-covariance matrix M_C is given by

$$\mathrm{M}_C = \mathbb{E}\left[CC^{\mathrm{T}}\right]. \quad (7)$$

Then, the covariance matrix K_D of the corresponding real random $2M$-vector D is given by

$$\mathrm{K}_D = \left[\begin{array}{c|c} \frac{1}{2}[\mathrm{Re}\{\mathrm{K}_C\} + \mathrm{Re}\{\mathrm{M}_C\}] & \frac{1}{2}[-\mathrm{Im}\{\mathrm{K}_C\} + \mathrm{Im}\{\mathrm{M}_C\}] \\ \hline \frac{1}{2}[\mathrm{Im}\{\mathrm{K}_C\} + \mathrm{Im}\{\mathrm{M}_C\}] & \frac{1}{2}[\mathrm{Re}\{\mathrm{K}_C\} - \mathrm{Re}\{\mathrm{M}_C\}] \end{array}\right]. \quad (8)$$

A circular-symmetric complex Gaussian (CSCG) random vector (Rvec) is defined as $\mathrm{M}_C = 0$. Then, K_D is represented by

$$\mathrm{K}_D = \left[\begin{array}{c|c} \frac{1}{2}\mathrm{Re}\{\mathrm{K}_C\} & -\frac{1}{2}\mathrm{Im}\{\mathrm{K}_C\} \\ \hline \frac{1}{2}\mathrm{Im}\{\mathrm{K}_C\} & \frac{1}{2}\mathrm{Re}\{\mathrm{K}_C\} \end{array}\right]. \quad (9)$$

Ⅳ. I/Q Channels

For I/Q channels, K_D should take the form

$$\mathrm{K}_D = \left[\begin{array}{c|c} \frac{1}{2}\mathrm{Re}\{\mathrm{K}_C\} & \mathbf{0} \\ \hline \mathbf{0} & \frac{1}{2}\mathrm{Re}\{\mathrm{K}_C\} \end{array}\right]. \quad (10)$$

because the I/Q channels are orthogonal. However, the covariance matrix K_D of the CSCG Rvec in (9) does not satisfy the covariance matrix K_D of the I/Q channels in (10).

Ⅴ. An Additional Condition for I/Q Channels

In this section, we present an additional condition for CSCG Rvecs to be valid over I/Q channels. This additional condition is given by

$$\mathrm{Im}\{\mathrm{K}_C\} = \mathbf{0}. \quad (11)$$

To have the equation in (11), the following condition should be satisfied

$$\rho_{i,j} = \mathrm{Re}\{\rho_{i,j}\}. \quad (12)$$

Therefore, the additional condition can be stated formally as follows:

The CCC $\rho_{i,j}$ of a CSCG Rvec should be purely-real over I/Q channels.

It should be noted that (12) implies $\mathrm{Im}\{\rho_{i,j}\} = 0$. Because $\mathrm{Im}\{\rho_{i,j}\}$ includes the correlation of the *i*th and *j*th users over I/Q channels, the condition $\mathrm{Im}\{\rho_{i,j}\} = 0$ effectively removes the correlation between orthogonal I/Q channels.

Note that a CSCG random variable (RV) completely models practical I/Q channels. This is because for a CSCG RV $\sqrt{P_A\beta_1}c_1$ of the first user,

$$\mathrm{K}_{\left[\mathrm{Re}\{\sqrt{P_A\beta_1}c_1\}\ \mathrm{Im}\{\sqrt{P_A\beta_1}c_1\}\right]} = P_A\beta_1 \left[\begin{array}{c:c} \frac{1}{2} & 0 \\ \hdashline 0 & \frac{1}{2} \end{array}\right], \quad (13)$$

and

$$\begin{aligned} \mathrm{K}_{\sqrt{P_A\beta_1}c_1} &= \mathbb{E}\left[\sqrt{P_A\beta_1}c_1\sqrt{P_A\beta_1}c_1^{\mathrm{H}}\right] \\ &= P_A\beta_1. \end{aligned} \quad (14)$$

Therefore, a CSCG RV $\sqrt{P_A\beta_1}c_1$ satisfies the additional condition in (11).

Ⅵ. Results and Discussions

To illustrate the impact of an invalid CCC, we assumed that the constant total transmitted signal-to-noise power ratio (SNR) was $P/\sigma^2 = 50$ and that the channel gain $|h_1|$ was $\sqrt{2}$. We compared the achievable data dates of the invalid CCCs (i.e., purely imaginary CCC $\rho_{i,j} = j0.819$ and non-purely real CCC $\rho_{i,j} = 0.819/\sqrt{2} + j0.819/\sqrt{2}$) to that of the valid CCC (i.e., purely real CCC $\rho_{i,j} = 0.819$). For a fair comparison, we set the absolute values of these three cases to be equal (i.e., $|\rho_{i,j}| = 0.819$), as shown in Fig. 1. Note that the achievable data rate of a purely imaginary CCC $\rho_{i,j} = j0.819$ is expressed by

$$R_1^{(\mathrm{SIC;\,CIS})} = \log_2\left(1 + \frac{|h_1|^2 P\beta_1\left(1 - |\rho_{1,2}|^2\right)}{\sigma^2}\right), \quad (15)$$

whereas the achievable data dates of the purely real CCC $\rho_{i,j} = 0.819$ and non-purely real CCC $\rho_{i,j} = 0.819/\sqrt{2} + j0.819/\sqrt{2}$ are given in (3).

As Fig. 2 shows, $R_1^{(\mathrm{SIC;\,CIS})}$ for $\rho_{i,j} = j0.819$ and $\rho_{i,j} = 0.819/\sqrt{2} + j0.819/\sqrt{2}$ were larger than $R_1^{(\mathrm{SIC;\,CIS})}$ for $\rho_{i,j} = 0.819$. It should be noted that $R_1^{(\mathrm{SIC;\,CIS})}$ for purely imaginary CCC

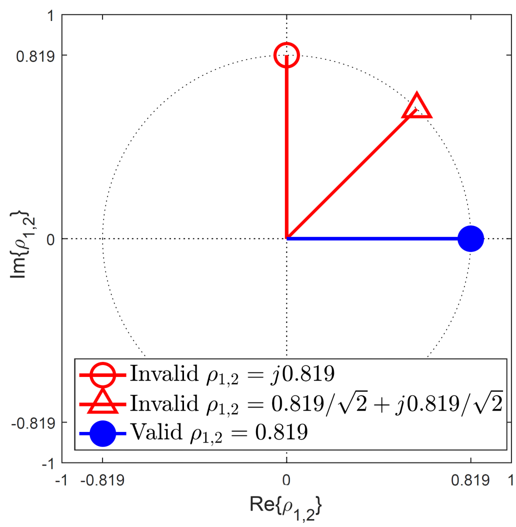

Fig. 1. An example for valid and invalid CCC $\rho_{1,2}$.

$\rho_{i,j} = j0.819$ was the largest among the three cases. The main point of this numerical result as given in Fig. 2 is that even though $R_1^{(\text{SIC; CIS})}$ for $\rho_{i,j} = j0.819$ and $\rho_{i,j} = 0.819/\sqrt{2} + j0.819/\sqrt{2}$ were larger than $R_1^{(\text{SIC; CIS})}$ for $\rho_{i,j} = 0.819$, $R_1^{(\text{SIC; CIS})}$ for $\rho_{i,j} = j0.819$ and $\rho_{i,j} = 0.819/\sqrt{2} + j0.819/\sqrt{2}$ could not be achieved over practical I/Q channels because $\rho_{i,j} = j0.819$ and $\rho_{i,j} = 0.819/\sqrt{2} + j0.819/\sqrt{2}$ cannot be implemented practically. Therefore, when we consider a CSCG Rvec over practical I/Q channels, we require an additional condition (i.e., purely real CCC $\rho_{i,j} = \text{Re}\{\rho_{i,j}\}$) along with the assumption of a CSCG Rvec.

In addition, in Fig. 3, we depict $R_1^{(\text{SIC; CIS})}$ versus the varying SNR P/σ^2, with the fixed power allocation $\beta_1 = 0.2$. As Fig. 3 shows, we observed the similar results as those presented in Fig. 2 (i.e., $R_1^{(\text{SIC; CIS})}$ for invalid $\rho_{i,j}$ is larger than $R_1^{(\text{SIC; CIS})}$ for valid $\rho_{i,j}$).

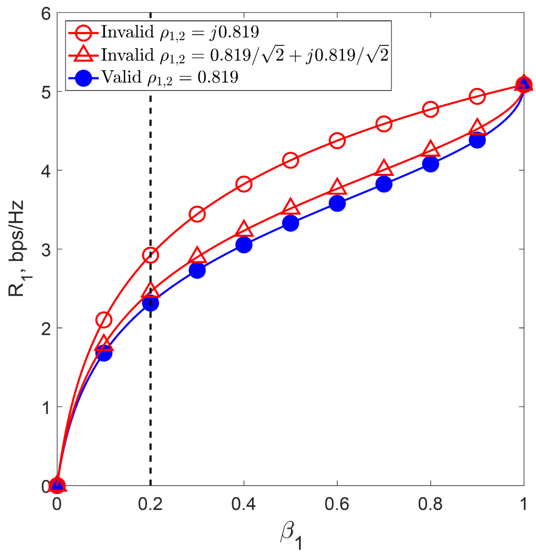

Fig. 2. Achievable data rates for valid and invalid CCC $\rho_{1,2}$.

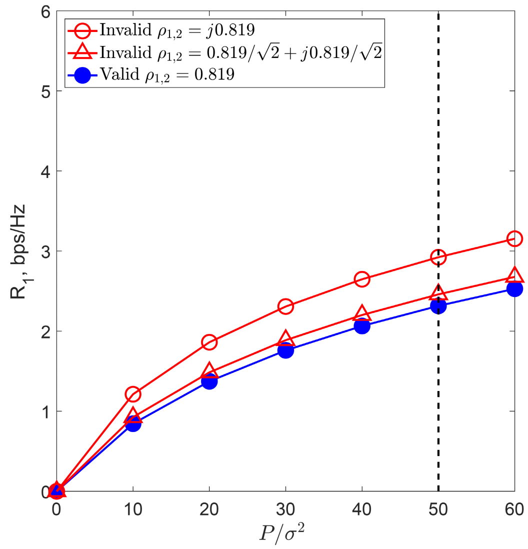

Fig. 3. Achievable data rates versus varying P/σ^2, with the fixed power allocation $\beta_1 = 0.2$, for valid and invalid CCC $\rho_{1,2}$.

Ⅶ. Conclusion

In this study, we proposed an additional condition for CSCG Rvec to be valid over practical I/Q channels. This constraint was given by a purely-real CCC, along with a zero complex pseudo-covariance matrix. We then investigated the effects of a purely-real CCC on the achievable data rate of a stronger channel user of NOMA with CIS, as compared with that of a general CCC. Results showed that when we work on I/Q channels, a purely-real CCC constraint should be assumed.

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