

## 저복잡도 LoRa 심볼 검파 방식

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## A Low-Complexity Symbol Detection Scheme for LoRa Signals

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## 요 약

IoT(Internet of Things) 기기를 광범위하게 이용하기 위해서는 통신기술을 저전력/저비용으로 구현하는 것이 중요하다. 이를 달성하기 위해, 본 논문에서는 IoT 분야에서 널리 이용되는 LPWA(low power wide area) 기술인 LoRa를 위한 심볼 검파 문제를 다룬다. 특히 IoT 기기의 저전력/저비용 구현을 위해, 저복잡도 LoRa 심볼 검파 방식을 제안한다. 제안된 방식의 동작을 검증하기 위해 기존 FFT(fast Fourier transform) 기반 심볼 검파 방식과의 성능비교를 수행하였고, 제안된 방식이 최적의 심볼 검파 성능을 보이면서도 계산량을 효과적으로 줄임을 보였다.

**Key Words** : Internet of Things (IoT), LoRa, symbol detection

## ABSTRACT

For the widespread use of Internet of Things (IoT) devices, a low-power low-cost implementation is important for the IoT communications. In order to achieve this goal, in this letter, we consider the problem of symbol detection for LoRa, which is one of the most widely-used low power wide area (LPWA) technologies for IoT. Especially for achieving long battery life and low implementation cost for IoT devices, a low complexity symbol

detection algorithm is proposed for the LoRa modulated signals. In order to evaluate the performance of the proposed scheme, compared to the conventional fast Fourier transform (FFT) based symbol detection, we show that the proposed scheme effectively reduces the computational complexity while achieving the optimal performance for symbol detection.

## I. Introduction

Recently, Internet of Things (IoT) is expected to provide intelligent and advanced services by connecting a wide range of devices to the Internet. For such massive Internet connection, low power wide area (LPWA) technologies play an important role, among which LoRa is the most promising, popular, and widely used one for IoT. In this case, it is of paramount importance to realize a low cost implementation and to prolong the battery life of the IoT devices, by reducing the system/hardware design complexity<sup>[1]</sup>. To achieve this goal, we propose a new and efficient symbol detection scheme for LoRa, which yields lower complexity than the conventional fast Fourier transform (FFT)-based LoRa symbol detection but achieves the optimal detection performance. Through our proposed algorithm, the symbol detection complexity is reduced by recursively performing time-domain decimation on the cross-correlation operation between the received signals and LoRa waveforms. This is the key technical and novel contribution of this letter.

## II. Conventional LoRa Symbol Detection

Let us consider a transmitter and a receiver employing the LoRa signalling scheme. The transmitted symbol  $a \in \{0, 1, \dots, 2^{\text{SF}} - 1\}$  is denoted by

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$$a = \sum_{k=1}^{\text{SF}} \mathbf{b}(k) 2^{k-1}, \quad (1)$$

where SF is an integer parameter which is usually called spreading factor in the spread-spectrum techniques,  $\mathbf{b}$  is a  $1 \times \text{SF}$  vector containing binary elements which are data bits to be transmitted, and  $\mathbf{b}(k) \in \{0, 1\}$  is the  $k$ -th element of the vector  $\mathbf{b}$ . Then, the LoRa waveform transmitted by the transmitter is given by<sup>[2]</sup>

$$x(n, a) = e^{j2\pi n \left( \frac{a}{M} - \frac{1}{2} + \frac{n}{2M} \right)}, \quad n = 0, \dots, M-1 \quad (2)$$

where  $n$  is a discrete time index and  $M = 2^{\text{SF}}$ .

In order to demodulate the received signal  $r(n) = x(n, a) + w(n)$  for  $n = 0, \dots, M-1$ , where  $w(n)$  is additive white Gaussian noise with zero mean, the receiver detects the symbol by choosing  $a$  that maximizes the correlation between the received signal  $r(n)$  and the LoRa waveform  $x(n, a)$  in (2)<sup>[2]</sup>. The detected symbol  $\hat{a}$  can be mathematically given as

$$\hat{a} = \arg \max_{a \in \{0, 1, \dots, 2^{\text{SF}} - 1\}} c(a), \quad (3)$$

where

$$c(a) = \sum_{n=0}^{M-1} r(n) x^*(n, a), \quad a = 0, \dots, M-1 \quad (4)$$

and  $(\cdot)^*$  is the complex conjugate operation. In LoRa systems, the correlation  $c(a)$  in (4) can be obtained by utilizing the FFT algorithm and the required number of multiplications is  $2^{\text{SF}} + 2^{\text{SF}-1} \log_2(2^{\text{SF}})$ <sup>[2,3]</sup>.

### III. Proposed LoRa Symbol Detection

In this section, we propose a low complexity symbol detection scheme for LoRa signals. The key idea of the proposed scheme is to reduce the number of multiplications to calculate the correlation  $c(a)$  in (4), which is obtained by  $\log_2 M$  stages.

Specifically, at each stage, we find out that the time-domain decimation can be applied to the LoRa signals  $x(n, a)$  in (4). Let us first look into the last  $\log_2 M$ -th stage. At the  $\log_2 M$ -th stage, for  $a = 0, \dots, M-1$ , the correlation  $c(a)$  in (4) can be expressed by the weighted sum of two correlation terms as follows:

$$c(a) = c_1^{(\log_2 M)}(a) \quad (5)$$

$$\begin{aligned} &= \sum_{n=0}^{\frac{M}{2}-1} r(2n) e^{-j2\pi(2n) \left( \frac{a}{M} - \frac{1}{2} + \frac{2n}{2M} \right)} + e^{-j2\pi \left( \frac{a}{M} - \frac{1}{2} + \frac{1}{2M} \right)} \\ &\quad \times \sum_{n=0}^{\frac{M}{2}-1} r(2n+1) e^{-j2\pi(2n) \left( \frac{a+1}{M} - \frac{1}{2} + \frac{2n}{2M} \right)} \end{aligned} \quad (6)$$

$$\begin{aligned} &= \sum_{n=0}^{\frac{M}{2}-1} r(2n) e^{-j2\pi(2n) \left( \frac{\frac{M}{2} \lfloor \frac{a}{M/2} \rfloor + \text{mod}(a, M/2)}{M} - \frac{1}{2} + \frac{2n}{2M} \right)} \\ &\quad + e^{-j2\pi \left( \frac{a}{M} - \frac{1}{2} + \frac{1}{2M} \right)} \sum_{n=0}^{\frac{M}{2}-1} r(2n+1) \\ &\quad \times e^{-j2\pi(2n) \left( \frac{\frac{M}{2} \lfloor \frac{a+1}{M/2} \rfloor + \text{mod}(a+1, M/2)}{M} - \frac{1}{2} + \frac{2n}{2M} \right)} \end{aligned} \quad (7)$$

$$\begin{aligned} &= \sum_{n=0}^{\frac{M}{2}-1} r(2n) e^{-j2\pi n \lfloor \frac{a}{M/2} \rfloor} e^{-j2\pi(2n) \left( \frac{\text{mod}(a, M/2)}{M} - \frac{1}{2} + \frac{2n}{2M} \right)} \\ &\quad + e^{-j2\pi \left( \frac{a}{M} - \frac{1}{2} + \frac{1}{2M} \right)} \sum_{n=0}^{\frac{M}{2}-1} r(2n+1) e^{-j2\pi n \lfloor \frac{a+1}{M/2} \rfloor} \\ &\quad \times e^{-j2\pi(2n) \left( \frac{\text{mod}(a+1, M/2)}{M} - \frac{1}{2} + \frac{2n}{2M} \right)} \end{aligned} \quad (8)$$

$$\begin{aligned} &= \sum_{n=0}^{\frac{M}{2}-1} r(2n) x^*(2n, \text{mod}(a, M/2)) + W(a, \log_2 M) \\ &\quad \times \sum_{n=0}^{\frac{M}{2}-1} r(2n+1) x^* \left( 2n, \text{mod} \left( a + D(\log_2 M), \frac{M}{2} \right) \right), \end{aligned} \quad (9)$$

where

$$W(a, i) = e^{-j2\pi \left( \frac{a}{2^i} - \frac{M}{2^{i+1}} + \frac{M}{2^{2i+1}} \right)}, \quad (10)$$

$$D(i) = \frac{M}{2^i}, \quad (11)$$

$$c_j^{(i)}(a) = \sum_{n=0}^{2^i-1} r\left(\frac{M}{2^i}n + \overline{2^i(j-1)}\right) x^* \left(\frac{M}{2^i}n, \text{mod}(a, 2^i)\right),$$

$$i = 1, \dots, \log_2 M \text{ and } j = 1, \dots, \frac{M}{2^i}, \quad (12)$$

$\lfloor \cdot \rfloor$  is the round-down operation,  $\text{mod}(x, y)$  is the remainder after dividing  $x$  by  $y$ ,  $i$  is the stage index, and  $\overline{x}$  is the bit-reversed value of  $x$ . The equality (a) is obtained by substituting (2) into (4) and splitting the right-hand-side terms of (4) into even/odd-indexed terms. The equality (b) is obtained by separating  $a$  and  $a+1$  into two terms, i.e.,  $a = M/2 \lfloor 2a/M \rfloor + \text{mod}(a, M/2)$  and

$$a+1 = \frac{M}{2} \left\lfloor \frac{a+1}{M/2} \right\rfloor + \text{mod}(a+1, M/2),$$

respectively. The equality (c) is obtained by simple manipulations. The equality (d) follows from the periodicity of the exponential function, i.e.,

$$e^{-j2\pi n \lfloor \frac{a}{M/2} \rfloor} = e^{-j2\pi n \lfloor \frac{a+1}{M/2} \rfloor} = 1.$$

The mathematical derivation in (5)-(9) is so-called the time-domain decimation, since the  $M$ -point correlation  $c(a)$  in (4) is splitted into the  $\frac{M}{2}$ -point correlation terms  $c_1^{(\log_2 M - 1)}(\text{mod}(a, M/2))$  and  $c_2^{(\log_2 M - 1)}(\text{mod}(a + D(\log_2 M), M/2))$  in (9), which are respectively for the even-indexed received signal  $[r(0), r(2), \dots, r(M-2)]$  and the odd-indexed received signal  $[r(1), r(3), \dots, r(M-1)]$ . In the same way, the time-domain decimation can be successively performed at the remaining stages, i.e.,

**Algorithm 1** Proposed algorithm to obtain  $c(a)$  in (4)

- 1: Initialize  $i := 1$ .
- 2: **repeat**
- 3:   **if**  $i = 1$  **then**
- 4:     Calculate  $c_j^{(1)}(a) = r(\overline{2j-2}) + W(a, 1)r(\overline{2j-1})$   
       for  $a = 0, 1$  and  $j = 1, \dots, \frac{M}{2}$ .
- 5:   **else**
- 6:     Calculate  $c_j^{(i)}(a) = c_{2j-1}^{(i-1)}(\text{mod}(a, 2^{i-1})) + W(a, i)$   
        $\times c_{2j}^{(i-1)}(\text{mod}(a + D(i), 2^{i-1}))$  for  
        $a = 0, \dots, 2^i - 1$  and  $j = 1, \dots, \frac{M}{2^i}$ .
- 7:   **end if**
- 8:   Set  $i := i + 1$ .
- 9: **until**  $i \leq \log_2 M$
- 10: Set  $c(a) = c_1^{(\log_2 M)}(a)$  for  $a = 0, \dots, M-1$ .

$$c_j^{(i)}(a) = c_{2j-1}^{(i-1)}(\text{mod}(a, 2^{i-1})) + c_{2j}^{(i-1)}(\text{mod}(a + D(i), 2^{i-1})) \times W(a, i).$$

The proposed algorithm using the above key idea is summarized in Algorithm 1, where it can be observed that the chirp correlation algorithm proposed in (5)-(12) can be constructed through  $\log_2 M$  stages, each of which has  $M$  weighted sums. In order to help understanding, for the case of  $M=8$ , the block diagram of Algorithm 1 is shown in Fig. 1, where three stages are performed and each stage has eight weighted sums. Note that Fig. 1 is not the butterfly structure which corresponds to the conventional FFT algorithm.

After obtaining  $c(a)$  by using Algorithm 1, the LoRa symbol can be detected as in (3). In order to evaluate detection performance of the proposed scheme, in Fig. 2, the bit error rate (BER) versus signal-to-noise ratio (SNR) is plotted for SF=7,...,12. As widely known in spread-spectrum communication systems, the BER decreases as the SF increases. Furthermore, since the value of the correlation  $c(a)$  computed by Algorithm 1 is the same as in (4), the BER of the proposed scheme is the same as that of

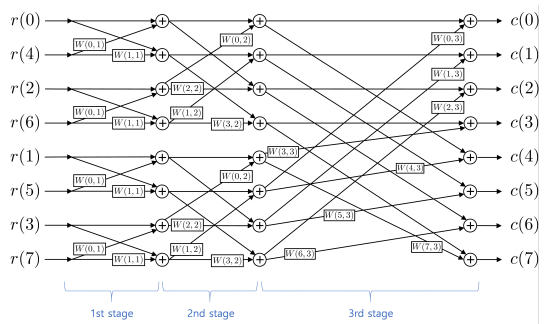


Fig. 1. Block diagram of Algorithm 1 with  $M=8$ .

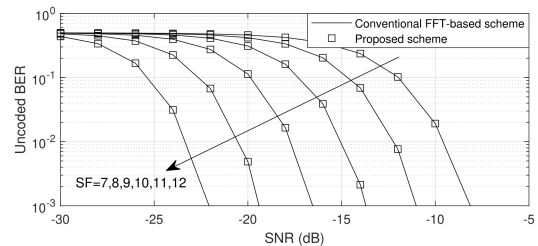


Fig. 2. Unencoded BER versus SNR.

the FFT-based optimal symbol detection<sup>[2,3]</sup>.

#### IV. Complexity Analysis and Conclusion

In this section, the symbol detection complexity is compared. The number of multiplications with the conventional FFT-based scheme is  $2^{\text{SF}} + 2^{\text{SF}} \log_2(2^{\text{SF}})$ <sup>[2,3]</sup>. On the other hand, with the proposed scheme in Algorithm 1, since each stage needs  $2^{\text{SF}}$  multiplications, the number of multiplications required to obtain  $\hat{a}$  in (3) is  $2^{\text{SF}} \log_2(2^{\text{SF}})$ . Therefore, the proposed scheme reduces the complexity by  $2^{\text{SF}}$ , i.e., the complexity is more effectively reduced for larger SF. This exponential complexity reduction may be significant for remotely located IoT devices, since the maximum communication range becomes longer as the SF increases and thus the high SF may be allocated to the devices located at a long distance.

We proposed a low complexity symbol detection scheme for LoRa. Interestingly, the symbol detection complexity was reduced while the optimal detection performance was achieved.

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